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# Inflation, Default, and the Currency Composition of Sovereign Debt in Emerging Economies

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## Abstract

In emerging market economies, governments issue debt denominated both in their own currency and in foreign currencies. I develop a theory of the optimal composition of sovereign debt between local and foreign currencies. In a model with a micro-founded monetary framework, a government controls monetary policy and has the ability to borrow from abroad using both local and foreign currency bonds. In this model, local currency bonds differ from foreign currency bonds in two important ways. Unlike foreign currency bonds, local currency bonds function as a contingent claim, allowing governments to more easily smooth consumption over time. In addition, the threat of strategic inflation limits the amount that a government can borrow using local currency bonds (but has no direct effect on foreign currency borrowing). When governments can issue both local and foreign currency bonds, equilibrium rates of inflation and national welfare are higher than when the government can issue only foreign currency bonds. In addition, I find that as the cost of default falls, governments choose to issue a larger proportion of debt in their local currency. Compared to monetary regimes that cannot commit to future actions, credible monetary policy commitments can eliminate the risk of strategic inflation and improve economic outcomes.

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# 1 Introduction

A proper analysis of sovereign borrowing behaviors demands consideration of the currency risk associated with sovereign debt. This is especially true for emerging and developing market economies whose governments, unlike those of developed nations, borrow substantial amounts in foreign currencies. In the past century, those countries have endured debt crises and hyper-inflationary episodes far more frequently than their developed nation counterparts. Given those observations, I have developed a theory of sovereign borrowing behaviors in emerging market economies that explores the optimal currency denomination of debt by generating endogenous currency risk and default risk.

Local and foreign currency debt differ in their risk profiles. Both types of debt are subject to default risk, the risk that the government will choose to default. However, unlike foreign currency debt, the payoff on local currency debt carries currency risk. If the local currency declines in value in foreign exchange markets, the real value of the payoff on local currency debt will fall.

Data on sovereign debt show substantial variation in the currency composition of that debt among different nations and over time for a large number of nations.[1] Figure 1 illustrates trends in debt denomination for a selection of emerging market (EM) economies.[2] In the years preceding the East Asian Crisis in 1998, EM sovereign debt portfolios shifted dramatically away from local currency debt. However, in the years following the crisis, that trend reversed. EM countries issued increasing amounts of local currency debt in the past decade, and that trend continues today. Nonetheless, as shown in Figure 2, EM governments generally still borrow less in their own currency than do governments of developed countries.

Historically, governments that borrow in local currency have used inflation to erode the

[1]See Cowan et al. (2006) for a comprehensive discussion of borrowing patterns in Latin American countries.

[2]I define local currency sovereign debt as fixed interest rate debt, denominated in a country's own currency, that is not indexed to inflation or an exchange rate. Governments do, indeed, issue local currency indexed debt. However, even though these bonds *are* denominated in local currency, their payoff functions more closely resemble those for foreign currency debt.

real value of their debt obligations. Reinhart and Sbrancia (2015) claim that inflation (in conjunction with financial repression) after World War II significantly reduced government debt burdens in countries such as the United States and the United Kingdom.[3] Although these possible episodes of strategic inflation may be identifiable in developed countries, EM countries have historically issued far less local currency debt. Additionally, that local currency debt, especially recently, has tended to be of short maturity, making it more difficult to employ this debt-reduction strategy. Nonetheless, nominal debt can and has been eroded through strategic inflation in the past. Therefore, inflation risk must certainly contribute to the pricing of nominal sovereign debt.

For this paper, I constructed a micro-founded dynamic sovereign debt model that explores the factors influencing the currency composition of sovereign debt. In the model, a government unable to commit to future policies borrows and prints money to improve the welfare of its constituency. When borrowing, the government can choose to issue bonds denominated either in the local currency or in a foreign currency. The domestic economy is constructed using the Lagos and Wright (2005) monetary framework (henceforth LW), providing a richer environment from which to study money demand and pricing dynamics.[4] In the model, the government has two tools to reduce the burden of international debt in the short term: inflation and default. By controlling the money supply, the government can inflate away the real value of nominal debt at the expense of domestic economic efficiency. Additionally, sovereigns can explicitly default on their debt, directly reducing debt obligations.

Empirical studies trace changes in the currency composition of sovereign debt to a variety of related factors. Claessens et al. (2003) find that the size of an economy, low inflation,

[3] According to Reinhart and Sbrancia, "financial repression includes directed lending to government by captive domestic audiences (such as pension funds), explicit or implicit caps on interest rates, regulation of cross-border capital movements, and (generally) a tighter connection between government and banks."

[4] Williamson and Wright (2010) justify the benefits of a micro-founded monetary model over the common reduced-form monetary models. Money search models create a richer monetary framework and generate a fundamental role of money through the model environment rather than preferences or a cash-in-advance constraint. In these models, money is essential because it allows agents to overcome trading frictions in situations with imperfect recordkeeping and incomplete insurance markets.

and capital account openness are all correlated with a higher proportion of local currency sovereign debt to total sovereign debt. Burger et al. (2012) produce similar results, showing that stable inflation and strong legal safeguards for creditors are linked to higher proportions of local currency sovereign borrowing. In their comprehensive study of Latin American debt markets, Borensztein et al. (2006) emphasize that while local currency sovereign borrowing has expanded in recent years, much of this increase comes from short-term bonds, which carry significant roll-over risk.[5] Hoschka (2005) highlights regional political initiatives, namely multilateral development banks such as the Asian Development Bank (ADB) and the Inter-American Development Bank (IADB) as possible key drivers behind growth in local currency sovereign borrowing. Furthermore, Spiegel (2009) emphasizes the role of market size in the growth of local currency bond markets and claims that increased financial liberalization and regulatory standards strengthen the development of local currency bond markets.

Decisions about fiscal policy, monetary policy, and the denomination of sovereign debt are all inextricably linked. Past monetary policy decisions affect the current borrowing capacity of governments (both the amount and denomination), which, in turn, influences future monetary policies. Therefore, any analysis of the composition of sovereign debt requires an understanding of how monetary and fiscal policy are co-determined. By endogenizing all three policy choices in my model, I developed a new theory to explain why we see such variation in the currency composition of sovereign debt.

Domestic monetary policy and domestic price dynamics are key determinants of the currency composition of emerging market sovereign debt.[6] Unlike with foreign currency bonds, the real burden of locally denominated bonds is determined by the domestic price level. When prices and the business cycle vary predictably, it will be in the best interest of the

[5]The authors note that a lack of monetary policy credibility may contribute to the short maturity length for local currency sovereign debt. The risk of inflation or currency crises inflates the cost of local currency borrowing at long horizons, making short-term local currency bonds more attractive to the issuing governments.

[6]I focus on small EM economies and assume that domestic shocks in these countries have no effect on prices in their developed country counterparts. More generally, I assume that purchasing power parity holds unconditionally as I do not explicitly model foreign exchange markets.

sovereign to take advantage of those dynamics. By issuing local currency bonds the government can better smooth its constituents' consumption over time than if the government could issue only foreign currency bonds. Yet, because locally denominated debt endows the government with the ability to alter the real payout structure of its obligations, there will be an inherent tension between the ex-ante desire to use the insurance aspect of these bonds and the ex-post desire of governments to create unexpected inflation to erode their worth. The costs of creating this inflation will therefore play a large role in the decision to borrow in the home currency.[7]

In this paper, I characterize some of the important trade-offs associated with an emerging economy's use of default and inflation in order to reduce debt. Upon calibrating my model to the Mexican economy, I found that even under extreme circumstances, in equilibrium, governments will resort to outright default only on foreign-denominated debt. For local currency bonds, erosion through unexpected inflation, while costly, is preferred. Countries will inflate away their debt only in bad times when repaying that debt would be especially costly. When the government has unlimited control over monetary policy, the equilibrium portfolio is skewed heavily toward foreign-denominated debt. With no restrictions on monetary policy actions, the incentive for an emerging market government to inflate away debt is too strong to support large amounts of domestic borrowing. However, monetary policy restrictions actually enable the government to borrow more in the local currency. Forcing a government to commit to an optimal monetary policy rule yields higher equilibrium local currency debt levels and higher aggregate welfare in equilibrium. In this way, the credibility of a government's monetary policy plays a key role in its decisions about debt denomination.

My paper builds upon the seminal Eaton and Gersovitz (1981) sovereign debt model, It incorporates both business-cycle shocks, as in Arellano (2008), and the debt denomination

[7]The LW monetary framework generates high inflation costs relative to reduced-form models. The high inflation costs stem from assumptions regarding the method of bargaining used in markets where money is a fundamental means of exchange. As discussed in Aruoba et al. (2007), alternative assumptions can significantly dampen these inflation costs.

decision.[8] A sizable body of literature exists that attempts to characterize the sovereign's debt denomination decision. Calvo (1978) first developed the idea that a government with nominal debt may have the ex-post incentive to use monetary policy to manipulate the real return on its nominal obligations. Models of nominal debt and discretionary monetary policy are subject to time inconsistency issues as discussed in Bohn (1988) and Persson et al. (2006) among others. Recent works have compared the characteristics of nominal and indexed sovereign debt to determine the optimal borrowing policies.[9] For example, Arellano and Heathcote (2010) compare a local currency debt regime and a dollarized regime and find that the dollarized regime may increase sovereign borrowing capabilities. Building upon these papers, I developed a model in which governments have access to both local and foreign currency debt at the same time. In a related paper, Araújo et al. (2013) extends Cole and Kehoe (1998) to explore the debt denomination decision in the presence of both inflation and default. Their paper finds that the correlation between the shock processes of the domestic and foreign countries play a large role in determining optimal borrowing decisions. My model extends their work by incorporating a robust monetary environment and by allowing governments to choose the amount of nominal debt to issue.

From the perspective of monetary economics, my paper is closely related to Martin (2011). In his paper, Martin embedded a sovereign debt model into the LW framework to analyze the tension between the incentive to erode the real value of nominal debt through inflation and the economic distortions that accompany inflation. In his model, the government finances the production of a public good through the sale of local currency bonds to domestic citizens. Unlike my paper, Martin, focuses on a closed economy and analyzes the trade-offs between domestic taxation and seigniorage revenue to finance public goods.[10] Martin concludes that the inability of the sovereign to commit to future policies creates fundamental economic distortions. Those same distortions are prevalent in my paper, in which the government

[8]Whereas the economies modeled in Eaton and Gersovitz (1981) and Arellano (2008) are endowment economies, I model a production economy with shocks to the productivity process.

[9]See Nicolini (1998), Díaz-Giménez et al. (2008) and Jahjah (2001).

[10]Bonds in Martin (2011) are locally denominated and not defautable.

borrows from external creditors and also has access to foreign currency debt.

## 2 Model

This paper explores the trade-offs between borrowing in local currency and foreign currency using a sovereign debt model that incorporates money demand. In that model, a small, open emerging market economy is subject to a stochastic process for domestic productivity. Presiding over that economy, a benevolent government maximizes the expected lifetime utility of its constituency using both fiscal and monetary policy. That government conducts monetary policy by controlling $g_t$, the growth rate of the aggregate money supply, $M_t$.[11] In conducting fiscal policy, the government has access to international debt markets where it can borrow from a pool of risk-neutral investors by issuing one-period bonds $b_{t+1}$ denominated in the domestic currency, pesos, or $b^*_{t+1}$, denominated in a foreign currency, dollars.[12] Costly inflation prevents the government from wanting to inflate away the value of its local bonds, and costly default discourages the government from explicitly defaulting on both local and foreign currency bonds.

The Lagos and Wright monetary framework provides some key advantages over more common monetary models that improve the model's ability to examine the debt denomination decision. Unlike money utility models, inflation is not only costly in terms of domestic utility, but also because it reduces domestic production. Through this channel, my model will be able to characterize the cost of inflation in terms of aggregate production. Additionally, LW improves on cash-in-advance models because the LW framework is able to better match empirical dynamics of money demand and prices, as discussed in Martin (2011). Using the LW framework to describe the monetary economy enables a dynamic and nuanced examination of the sovereign's choice of the currency denomination of its debt. For these

[11] The government chooses the aggregate amount of currency available in the economy in the following period $M_{t+1}$ through its choice of $g_t$ such that $M_{t+1} = g_t M_t$. The aggregate money supply plays a key role in facilitating transactions in the domestic economy.

[12] Asterisks in my model will indicate variables denominated in a foreign currency.

reasons, I employ the LW money demand framework in my model rather than other methods of modeling monetary economies.

## 2.1 Domestic Economy

My model extends the LW framework to incorporate stochastic labor productivity and variable money growth rates while embedding that framework into a sovereign debt model. A set of infinitely lived agents on the unit interval maximizes a stream of utility discounted at a rate $\beta \in (0, 1)$. Each period comprises two separate and sequential exchanges: a decentralized market (DM) followed by a centralized market (CM).

The market mechanism and consumer preferences in the decentralized market necessitate the use of money to facilitate production and consumption. Money is valuable in this model because agents in the DM produce a specific type of good but desire to consume a different type of good. This modeling framework attempts to replicate the fact that most transactions are not conducted through bartering. In other words, people use money to facilitate trade in transactions that occur between two agents where one of those agents has no desire to consume the specific good produced by the other agent.

In the DM, each agent is randomly paired with one other agent. With probability $\sigma_m$, an agent will produce the exact good $x$ that her match wishes to consume, but not vice versa. To rule out barter, I assume no meetings with a double coincidence of wants and, thus, the probability that any pairing produces no trade opportunity is $1 - 2\sigma_m$. Following the DM, all agents meet in a frictionless CM where entrants can produce and consume a homogeneous good $X$. Goods in both subperiods are assumed to be nonstorable. Because agents in the DM cannot commit to repay debt or keep reliable records in the CM, a medium of exchange, money, is necessary to facilitate DM trade. DM buyers will compensate DM sellers for their production with $\mu_t$, a monetary payment. Those monetary payments can then be used by DM sellers to purchase goods in the second market, the centralized market.

Those specifications, while complex, yield an environment that replicates the essential function of money. Unlike other models of money usage, the Lagos and Wright framework provides an environment in which money directly facilitates the transfer of goods from buyers to sellers. Without money (and with costly recordkeeping), agents in the economy would be unwilling to trade and would therefore resort to barter. However, because a coincidence of wants is unlikely, money enables consumers to compensate producers with a valuable asset in the absence of a coincidence of wants. In the LW economy, money is essential. The existence of money as a tradable asset enables more production and consumption, and thus higher welfare.

Both goods $x$ and $X$ are produced using only labor ($h$ and $H$, respectively) as an input. In the beginning of each full period, the economy receives a stochastic productivity shock $A$ such that one unit of labor input can produce $A$ goods in both subperiods.[13] Consumers in the DM derive utility $u(x)$ from consumption of the nonstorable DM good, and producers incur a utility cost of labor $c(h)$. In the CM, consumption of the homogeneous good yields utility $U(X)$ and production of the homogeneous good incurs a utility cost of $C(H)$. As in LW, preferences are assumed to be quasi-linear in CM labor with $C(H) = H$. The aggregate period utility function $u(x) - c(h) + U(X) - C(H)$ is assumed to be additively separable in its components. An agent enters the period with $m_t$ money holdings. The distribution of money holdings across agents in the economy is denoted by $F_t(m_t)$.

Agents in the model know all relevant state variables in the beginning of each period. The aggregate state of the economy I denote as $S_t = \{A_t, b_t, b_t^*, F_t\}$. All agents maximize their discounted lifetime utility given their own money holdings $m_t$ and the state of the world $S_t$, subject to a government policy function that I denote as $\Omega$. The amount of money held by the agent's counter-party in a match is labeled $\tilde{m}$ and the value function of an agent entering period $t$ as $V(m_t, S_t; \Omega)$:

[13]Thus, $x = f(h) = Ah$ and $X = f(H) = AH$.

$$
\begin{aligned}
V(m_t, S_t; \Omega) &= \sigma_m \int \{u\left[x_t(m_t, \tilde{m}_t, S_t; \Omega)\right] + W\left[m_t - \mu_t(m_t, \tilde{m}_t, S_t), S_t; \Omega\right]\} \, dF_t(\tilde{m}) \\
&+ \sigma_m \int \left\{-c\left[\frac{x_t(\tilde{m}_t, S_t; \Omega)}{A_t}\right] + W\left[m_t + \mu_t(\tilde{m}_t, m_t, S_t), m_t, S_t; \Omega)\right]\right\} dF_t(\tilde{m}) \\
&+ (1 - 2\sigma_m) W(m_t, S_t; \Omega).
\end{aligned}
$$

The value function is the weighted average of the three possible decentralized market outcomes: successful match as a buyer, successful match as a seller, and no match. Potential buyers and sellers who are successfully matched bargain over the potential surplus from their exchange. I assume a Nash Bargaining solution to this problem wherein the buyer has relative bargaining power $\theta \in (0, 1)$ and a threat point equal to the continuation value when no exchange occurs. The continuation value $W(m_t, S_t; \Omega)$ describes the value of entering the *centralized market* with money holdings $m_t$ with aggregate state $S_t$ and policy function $\Omega$. In each decentralized market meeting, production $x_t$ and the monetary transfer $\mu_t$ are chosen through Nash Bargaining:

$$
\max_{x_t, \, \mu_t} \; \left[u(x_t) + W(m_t - \mu_t, .) - W(m_t, .)\right]^{\theta} \left[-c\left(\frac{x_t}{A_t}\right) + W(\tilde{m}_t + \mu_t, .) - W(\tilde{m}_t, .)\right]^{1-\theta}. \quad (1)
$$

The Nash Bargaining problem is subject to four constraints. First, the monetary transfer between buyer and seller must be feasible $\mu_t \leq m_t$, meaning that the amount transferred can be no more than the buyer's initial money holding. Second, DM producers cannot produce negative amounts of the DM good or $x_t \geq 0$. Third, the amount of utility derived from DM consumption cannot be less than the utility value of the CM goods that could have been purchased with the money used as payment in the DM. Fourth, similarly, the utility cost of production in the DM cannot exceed the utility derived from the seller's purchases of CM goods using the money earned from DM trade. The price of CM goods is $p_{c,t}$, and I define

$\phi_t \equiv \dfrac{1}{p_{c,t}}$.

Upon completing DM transactions, agents enter the CM to produce and exchange homogeneous goods. In this second subperiod, agents maximize their utility by choosing labor hours $\bar{H} \geq H_t \geq 0$, consumption $X_t$ and money purchases $m_{t+1}$ subject to their budget constraint.

The government provides agents with a lump-sum transfer $T_t$ (or tax, if negative) comprising net revenues from external borrowing and money growth (in terms of CM goods). If the government's net borrowing revenue is negative (and the government creates no seigniorage revenue), $T_t$ serves as a direct tax on CM production. The same type of tax on money holdings occurs when the government decides to contract the money supply $g_t < 1$. It is through this transfer (or tax) that government actions directly influence the agent's budget constraints.

The household's CM problem is:

$$\begin{aligned} W(m_t, S_t; \Omega) \quad &= \quad \max_{m_{t+1},\, H_t,\, X_t} \left\{ U(X_t) - C(H_t) + \beta E_t \left( V(m_{t+1}, S_{t+1}; \Omega) \right) \right\} \\ s.t. \quad & X_t = A_t H_t + \phi_t m_t - \phi_t m_{t+1} + T_t. \\ & 0 \leq H_t \leq \bar{H}. \end{aligned}$$

Substituting $C(H_t) = H_t$ into the above equation yields:[14]

$$W(m_t, S_t; \Omega) \;\; = \frac{\phi_t m_t}{A_t} + \frac{T_t}{A_t} \;\; + \max_{m_{t+1},\, X_t} \left\{ U(X_t) - \frac{X_t}{A_t} - \frac{\phi_t m_{t+1}}{A_t} + \beta E_t \left( V(m_{t+1}, S_{t+1}; \Omega) \right) \right\}.$$

The assumption of quasi-linearity in CM labor utility has important implications for the model. First, the agent's value function will be linear in CM money holdings. In addition, all agents, in equilibrium, will optimally choose the same level of CM consumption

[14] Assume that $\bar{H}$ is sufficiently high to avoid a binding labor constraint.

$X^{opt}$ irrespective of their money holdings. This efficient quantity $X^{opt}$ is the consumption level that satisfies $U'(X_t) = (\frac{1}{A_t})C'(\frac{X_t}{A_t}) = \frac{1}{A_t}$, the first-order condition with respect to CM consumption. Both of these results will prove important in characterizing a type of equilibrium in which the distribution of money $F_t(m)$ is degenerate. Finally, note that the choice of $m_{t+1}$ does not depend on current money holdings.

## 2.2 Domestic Economy Equilibrium

Deriving a solution to this model follows the same procedure as in Williamson and Wright (2010). For the sake of brevity, this derivation can be found in the appendix.

If the domestic economy were controlled by a central planner, DM producers would always produce $x_t^{opt}$, the value of $x_t$ that maximizes total surplus and satisfies $(\frac{1}{A_t})c'\left(\frac{x_t}{A_t}\right) = u'(x_t)$. However, competitive Nash Bargaining often renders $x_t^{opt}$ an infeasible equilibrium outcome. The general solution to the Nash Bargaining problem can be described by the following equation, where the function $z(x_t, A_t)$ describes the seller's share of the surplus created from a successful DM match:

$$\frac{\mu_t \phi_t}{A_t} = z(x_t, A_t) \equiv \frac{\theta u'(x_t)c(\frac{x_t}{A_t}) + (1-\theta)(\frac{1}{A_t})c'(\frac{x_t}{A_t})u(x_t)}{\theta u'(x_t) + (1-\theta)(\frac{1}{A_t})c'(\frac{x_t}{A_t})}. \tag{2}$$

As a result of Nash Bargaining and quasi-linear preferences in the CM, the seller's share of DM surplus will be equivalent to the utility value (in terms of CM consumption goods) of the money received as payment for producing DM goods. Each peso transferred can be used to purchase $\phi_t$ goods in the CM market, thus saving DM sellers from having to work $\dfrac{\phi_t}{A_t}$ hours during the CM. When $\theta = 1$, DM consumers have full bargaining power and make a take-it-or-leave-it offer to the producers equal to their cost of production, $c(\frac{x}{A})$. Alternatively, when $\theta = 0$, consumers pay for their consumption with a monetary payment valued at the total utility attained through DM consumption. In equilibrium, consumers will compensate producers for their labor by paying them $\dfrac{\mu_t \phi_t}{A_t} = z(x_t, A_t)$. This condition equates the utility value of DM receipts (in terms of CM labor utility) to the seller's share of the surplus created from a DM match.

Nash Bargaining in the LW framework creates a hold-up problem that prevents a DM equilibrium in which sellers produce $x_t^{opt}$. Recall the necessary consumer optimality constraint $u(x_t) \geq \frac{\phi_t \mu_t}{A_t}$. If $u(x_t^{opt}) < \frac{\phi_t \mu_t^{opt}}{A_t}$, the DM consumer will decide not to trade all of her money with the DM producer. Restricting the amount of money buyers bring with them into a DM match ensures that the amount of utility derived from the DM match does not exceed the utility value of the money used to pay for DM goods. For most commonly used utility and cost functions, there exists some $\bar{x}_t \leq x_t^{opt}$ and an associated $\bar{\mu}_t \leq \mu_t^{opt}$ that satisfies $u(\bar{x}_t) = \frac{\phi_t \bar{\mu}_t}{A_t}$ representing the equilibrium DM outcome.

During the CM, agents work, trade money for goods and decide how much money to bring into the following period. Prices are determined through this optimization yielding the following equilibrium condition:

$$\frac{\phi_t}{A_t} = \beta E_t \left[ \left( \frac{\phi_{t+1}}{A_{t+1}} \right) \left( \sigma_m \mu_m(m_{t+1}, .) \left[ \left( \frac{u'(x_{t+1})}{z_x(x_{t+1}, A_{t+1})} \right) - 1 \right] + 1 \right) \right]. \tag{3}$$

This equation describes the money demand equilibrium in the centralized market. The utility value of spending a unit of currency (pesos) in the CM must be equal to the discounted value of holding onto that currency into the following period. In the next period, with probability $\sigma_m$, the agent will be a buyer in the next DM. As a buyer, an extra peso has a value equal to $\left( \frac{\phi_{t+1}}{A_{t+1}} \right) \mu_m(m_{t+1}, .) \left[ \left( \frac{u'(x_{t+1})}{z_x(x_{t+1}, A_{t+1})} \right) - 1 \right]$, the marginal utility of spending that currency in the DM minus the opportunity cost of spending that money in the DM. If the agent either does not get to be a buyer or decides not to bring that peso into the DM, then the marginal value of an extra peso will simply be equal to $\left( \frac{\phi_{t+1}}{A_{t+1}} \right)$, the utility value of one extra peso in terms of future CM utility. In a monetary equilibrium, $\phi_t$ will adjust to ensure that the money markets clear, meaning that $m_{t+1} = M_{t+1}$ and that there exists no opportunity for intertemporal arbitrage.

## 2.3 The Sovereign

The benevolent government acts to maximize the lifetime utility of its constituency. To do so, the government controls both monetary and fiscal policy.[15] The stochastic nature of the productivity process provides the government with incentive to engage in intertemporal consumption-smoothing. In periods when the productivity realization is low, the government can supplement the consumption of the household (in terms of CM goods) by selling bonds on the international bond market. In addition, the government has the power to print money and collect seigniorage revenue. Specifically, the government controls the gross money growth rate $g_t$ such that $M_{t+1} = g_t M_t$, where $M_t$ is the total money supply in the current period. Both varieties of government revenue (bond and seigniorage) are distributed to the household in the form of $T_t$, a lump-sum transfer during the CM. The sovereign publicly announces the size of this transfer at the beginning of every period. All bond-related transfers are completed in the centralized market and are written as contracts in terms of CM prices.

The sovereign's budget constraint is then:

$$T_t = \phi_t \left[ (1 - d_t) \left( q_t b_{t+1} - b_t \right) - d_t \gamma b_t \right] + \phi_t^* \left[ (1 - d_t^*) \left( q_t^* b_{t+1}^* - b_t^* \right) - d_t^* \gamma b_t^* \right] + \frac{(g_t - 1) M_t}{p_t}. \quad (4)$$

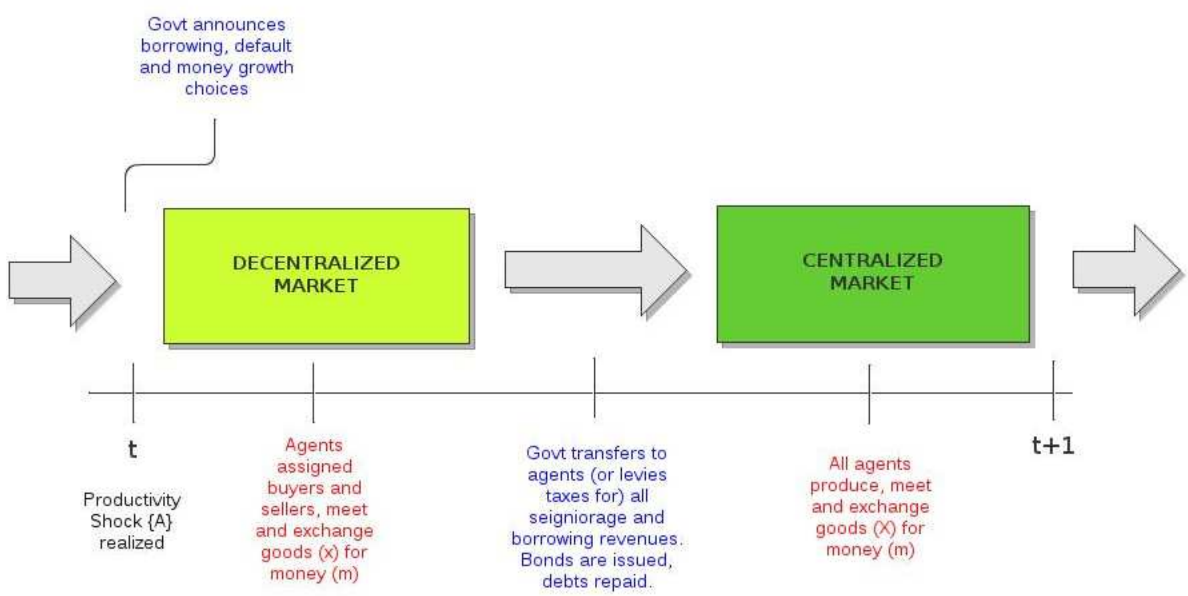
The sovereign has the power to sell one-period nominal bonds denominated both in the domestic currency (pesos) $b_{t+1}$ and in a foreign currency (dollars) $b_{t+1}^*$. Domestic (foreign) denominated bonds sell for price $q_t$ ($q_t^*$) and pay 1 peso (dollar) in the proceeding period.[16] Real bond revenues $\phi_t q_t b_{t+1}$ ($\phi_t^* q_t^* b_{t+1}^*$), therefore, depend crucially on the price levels of both the home and foreign countries. Default ($d = 1$) indicates whether the government decides to explicitly default on either type of debt. In the case of default, the government repudiates

[15] While advanced economies tend to have independent central bank authorities, evidence shows that in developing countries, fiscal and monetary policymakers may be more closely tied. Crowe and Meade (2008) find that even though EM economies have given more independence to their central banking authorities (and, to a lesser degree, transparency) since the 1980s, the measures of their central bank independence still generally lag behind those of advanced economies.

[16] I assume that all bond contracts are written in terms of centralized market prices.

some exogenously set $1-\gamma$ percentage of its debt obligation. The upper limit on CM labor hours $\bar{H}$ implies a natural borrowing limit equal to $\frac{\beta}{1-\beta}\bar{H}$. A timeline of government and household activities is displayed below.

The foreign country is assumed to be a developed nation with perfectly stable prices. Without loss of generality, I assume that the foreign price level (for its centralized market) follows a deterministic path where $\phi_t^* = 1$ for all $t$. Under the assumptions of purchasing power parity, the nominal exchange rate $e_t$ (in terms of dollars per peso) is simply the ratio of the domestic price level to the foreign price level. The exchange rate $e_t = \frac{p_{c,t}}{p^*_{c,t}} = p_t$ is thus directly proportional to the price level.[17] Any increase in peso prices can be seen as a devaluation of the peso and any decrease in prices will increase the peso's value relative to the dollar.




Through $T_t$, the government transfers all bond and seigniorage revenue back to the household. If debt obligations exceed bond revenue, $T_t$ may be negative. Under this scenario, the

[17] Simplifying exchange rate dynamics in this way implies that purchasing power parity holds unequivocally.

government taxes the household to pay off its deficit. If this tax becomes too burdensome, the government can either default on its debt or generate seigniorage revenue by printing money.

Choosing how exactly to model default is a problematic issue, discussed at length in the sovereign default literature.[18] Generally, there are two fundamental modeling choices to make: the costs of default and the benefits of default for the sovereign. Beginning with the costs of default, it is clear given the existence of sovereign borrowing that there must exist some appreciable cost to default. If default were costless, governments would always choose to default, thereby making borrowing unsustainable.[19] In addition, given that there are default costs, these costs must be sufficiently punitive to support the volume of debt observed in the data.

Recent sovereign debt models typically identify default costs as a direct loss of output, or alternatively as the financial autarky resulting from exclusion from debt markets (or sometimes a combination of both). An effective threat of exclusion requires credible collusion among lenders (Wright, 2005). Even if collusion is credible, the duration of credit market exclusion must be sufficiently long and costly to support lending and deter default. Empirically, defaulting countries resume foreign borrowing very soon after a default episode, sometimes less than a year after the default event.[20] The absence of observed borrowing during a default episode, the main evidence in favor of exclusion as a default cost, may be explained not by a collusive refusal to lend by financiers but by a sovereign's refusal to borrow at the offered interest rates.

I choose to penalize default through a direct linear utility penalty. This penalty may be interpreted as a proxy for some of the default costs suggested by the empirical literature: loss of wealth from domestic citizens and firms holding government debt or an increased

[18]For an extensive discussion on the empirical costs of default, see Borensztein and Panizza (2009).

[19]Bulow and Rogoff (1989) note that reputation costs alone cannot support borrowing in EMs.

[20]Gelos et al. (2011) determined that most countries defaulting in the 1980s resumed international borrowing 4 years after their initial default.

cost of trade credit for export-oriented firms.[21] Like the specification in Arellano (2008), my default penalty is relatively more costly for countries experiencing positive shocks.[22] When the sovereign chooses default, agents are levied a direct utility cost $\tau$ that is linearly deducted from the household utility function $V(.)$. When utility is high, this cost will be relatively more costly in terms of consumption than when utility is low.

I eschew output penalties for a variety of reasons. First, under standard preferences, linear utility penalties provide a simple way to make default more costly when utility is high than it is when utility is low. In addition, penalizing output has implications during the calibration process. To properly estimate the output/productivity process, one would have to adjust the data for the assumed output cost of default. Lastly, studies of the costs of sovereign default such as Levy-Yeyati and Panizza (2011) have established that default events typically follow a large fall in output, but they occur in periods when output is actually rising.[23] For these reasons, I have chosen to model the costs of default as a utility penalty.

Models of sovereign debt dating back to Eaton and Gersovitz (1981) specify various methods of default. Some papers describe default as a full repudiation of debt obligations, others model default as a partial reduction and still others describe the sovereign default process as a negotiation between the government and its creditors. Sturzenegger and Zettelmeyer (2008) find that the size of debt haircuts varies substantially among default episodes.[24] Benjamin and Wright (2009) find that bargaining delays and commitment issues can explain the large inefficiencies that arise from debt renegotiations. In comparison, Yue (2010) endogenizes debt recovery rates and finds that the recovery rate is negatively related to the level

[21] See Rose (2005) and Martinez and Sandleris (2011) for evidence supporting trade-related costs of default. Mendoza and Yue (2012) develop a model where default gives rise to trade frictions, generating default costs endogenously.

[22] In Arellano (2008), default is penalized through an output reduction when output is over some threshold. When a country defaults, output falls to some level $y$ when $y_t > y$ and faces no penalty when $y_t \leq y$. This threshold is calibrated for the Argentine economy and serves to ensure that default is relatively more costly when output is at high levels.

[23] Tomz and Wright (2007) conclude that the relationship between output and sovereign default events is negative, although weakly so.

[24] Jorgenson and Sachs (1988) show that present-value losses suffered by investors in the 1930s as a result of Latin American defaults varied from 37 percent in Columbia to 92 percent in Bolivia.

of sovereign debt. Given the variety of explanations for the important factors determining debt haircuts, I will remain agnostic to the determinants of haircut size and exogenously set a haircut percentage.[25]

My method of modeling the outcome of a default decision is as follows: When the government chooses to default, investors are forced to take a haircut of some $(1-\gamma)$ percent of their real debt holding. The remaining $\gamma$ percent of debt must, however, be repaid in the current period. In periods of default, the government is unable to borrow in the bond market where the government is in default (i.e. local currency or foreign currency). After the government is excluded from bond markets for one period, it can return to the market without any incoming debt and borrow as it chooses.[26] In the period after a dollar bond default, the country will enter the next period without any dollar-denominated obligations. A default on dollar bonds affects dollar-denominated borrowing but will not *directly* influence the market for peso-denominated debt. The same rules apply for default on peso-denominated debt.

The markets for peso-denominated and dollar-denominated debt are segmented, allowing the sovereign to selectively default on each type of debt separately. For example, the sovereign may choose to default only on dollar debt ($d_t^* = 1$) and maintain payment on its peso debt ($d_t = 0$), or vice versa. It is possible that both dollar and peso debt may be defaulted upon simultaneously; however, this need not be the case. Selective default is not uncommon among defaulting nations.[27] Therefore, in my model, I allow the government the choice of which bonds to honor and which to deny repayment.

In addition to explicit default, the government can erode the real value of peso-denominated debt by creating inflation. Controlling the money growth rate allows the sovereign to retain control over the domestic price level. Through this channel, the government can effectively

[25]The term *haircut* refers to the percentage of the present value of a bond that gets forgiven in a restructuring.

[26]Note that because the sovereign enters the market without any debt, it pays the same interest rate as would a government that enters the period without any debt and did not default in the previous period.

[27]One recent instance of selective default was the Ecuadorian default of 2008. In this episode, Ecuador refused payment on only a specific long-term debt issue held primarily by external creditors. Ecuador decided to return to international bond markets in 2014 and had little trouble selling its bond issue at a 7 percent interest rate.

erode the real value of its peso-denominated debt obligations through unexpected inflation. However, this inflation is not costless. As shown in equation (3), increasing the money supply directly affects the size of the monetary transfer between buyers and sellers. An announcement of a high rate of money growth necessarily devalues all current money holdings and makes sellers less willing to accept money as payment for DM production. In this way, inflation necessarily reduces model efficiency by exacerbating the hold-up problem. If the government were able to commit to future money growth rates, it could avoid creating the inefficiency that would make borrowing in the local currency cheaper.

## 2.4 Foreign Investors

Risk-neutral foreign investors lend to the government through purchases of one-period bonds. In addition to sovereign bonds, these investors have access to risk-free bonds paying the real rate of interest $r^f$. Perfect competition in the international bond market ensures that sovereign bonds are priced such that investors expect zero economic profit from their purchase. This result generates the following price functions for dollar-denominated and peso-denominated sovereign debt, respectively:

$$\phi_t^* q_t^*(A_t, b_{t+1}, b_{t+1}^*) b_{t+1}^* = \frac{1}{1+r^f} E_t \left[ \phi_{t+1}^* \left\{ (1-d_{t+1}^*) b_{t+1}^* + d_{t+1}^* \gamma b_{t+1}^* \right\} \right], and \tag{5}$$

$$\phi_t q_t(A_t, b_{t+1}, b_{t+1}^*) b_{t+1} = \frac{1}{1+r^f} E_t \left[ \phi_{t+1} \left\{ (1-d_{t+1}) b_{t+1} + d_{t+1} \gamma b_{t+1} \right\} \right]. \tag{6}$$

Real bond revenues lent to the sovereign today is equal to discounted expected real bond payment made by the sovereign during the next period. Because of their one-period term to maturity, bond prices in this model are not dependent on past borrowing behavior. When pricing bonds, the only relevant factors are the sovereign's current borrowing choice and the economy's current productivity draw. Investors require only those two pieces of information to calculate the two relevant factors for bond returns: expected default probabilities and expected inflation rates. Therefore, two governments with the same productivity $A$—one entering the period with no debt whatsoever and another coming into the period with a debt-to-GDP ratio of 1.00—both face the same bond price schedule.[28] The probability of default does not depend on a country's indebtedness today—only its productivity and indebtnedness tomorrow. In the model, a country with high debt today is likely to borrow more today than the country with no debt. However, assuming both types of countries had the same productivity level and wanted to issue the same amount of bonds, they would always face the same interest rate schedule because at every debt level they have an identical probability of default in the following period.

## 2.5 Normalization

I normalize all nominal variables to ensure stationarity of my model solution to avoid needing to keep the aggregate money supply as a state variable. This normalization both simplifies computation and presents clearly the role of money growth $g_t$ in determining the model equilibrium. All nominal variables are normalized by the money supply:

$$\hat{\phi}_t = \phi_t M_{t+1} \qquad \hat{m}_t = \frac{m_t}{M_t} \qquad \hat{\mu}_t = \frac{\mu_t}{M_t} \qquad \hat{b}_t = \frac{b_t}{M_t}.$$

Equation (3) can be rewritten as:

$$\frac{\hat{\phi}_t}{A_t} = \beta E_t \left[ \left( \frac{\hat{\phi}_{t+1}}{g_{t+1} A_{t+1}} \right) \left( \sigma_m \mu_m(m_{t+1}, .) \left[ \left( \frac{u'(x_{t+1})}{z_x(x_{t+1}, A_{t+1})} \right) - 1 \right] + 1 \right) \right]. \tag{7}$$

This equation is used to solve for $\hat{\phi}_t$, the normalized CM price level. This price can then be applied to the equilibrium conditions for the DM. In the DM, consumers enter the period with either $m_t < \bar{\mu}_t$ or $m_t \geq \bar{\mu}_t$. In the first case, the consumer is constrained by her money holdings, chooses $\hat{\mu}_t = 1$, and brings all of her money into the decentralized market.

[28]If this model allowed for the issuance of bonds with longer maturities, that condition would not hold. With long-term bonds, bond prices would be functions of both the productivity shock and the existing debt position.

Applying normalized variables to equation (2), the bargaining solution is given by:

$$\frac{\hat{\phi}_t \hat{\mu}_t}{A_t g_t} = \frac{\hat{\phi}_t}{A_t g_t} = z(x_t, A_t) \leq z(\bar{x}_t, A_t).$$

The above equation illustrates the effect of money growth on the equilibrium of the decentralized market. When contracting the money supply, the government increases the value of the money that consumers hold, increasing their purchasing power and equilibrium DM production. Conversely, creating inflation will lower the value of the DM consumer's money holdings, decreasing the equilibrium DM production level and reducing the economy's total surplus. In a world without any other considerations except maximizing total economic surplus, the government chooses to contract the money supply sufficiently to support $\bar{x}_t$, the highest feasible Nash bargaining equilibrium production level. Because an upper limit on DM production exists, a limit at which money supply reductions no longer induce more production similarly exists. This limit I define as $\bar{g}_t$:

$$\bar{g}_t \equiv \frac{\hat{\phi}_t}{A_t z(\bar{x}_t, A_t)}.$$

If the government chooses $g_t < \bar{g}_t$, money becomes so valuable that consumers begin to limit the amount of money they bring into DM meetings. As $g$ decreases further, $\hat{\mu}_t$ falls in tandem. The equilibrium of the decentralized market is then defined by the following equation:

$$\frac{\hat{\phi}_t \hat{\mu}_t}{A_t g_t} = z(\bar{x}_t, A_t).$$

The normalized equilibrium condition conveys (see the appendix for derivation) the relationship between monetary policy and economic efficiency. Money growth and inflation have real implications for the economy and may exacerbate the hold-up problem faced by agents in the decentralized market. However, by making today's currency less valuable relative to future currency, the government can decrease $\frac{\hat{\phi}_t \hat{b}_t}{g_t}$, the real burden of its domestic bonds coming due. This presents the fundamental tension in this model. In the presence of domestically denominated debt, reducing real payments on domestic debt through unexpected inflation may come at the cost of economic efficiency.

# 3 Solution Method

A full definition of the model's equilibrium can be found in the appendix.[29] To solve for that equilibrium, I solve a finite horizon problem, the limit of which converges to the infinite horizon solution; and to preserve the value of money in the final period $T$, I assume that households derive utility from money holdings in period $T+1$. In addition, in order for this game to support equilibrium lending, I assume that in period $T$ the sovereign must always pay its outstanding debt less the default haircut. Without this assumption, the sovereign would always default in the final period making borrowing in the penultimate period unsustainable. In addition, I assume some fixed continuation value for money holdings in period $T+1$. These requisite specifications allow me to solve the model. As I iterate backward, the effects of these assumptions will be discounted such that they are inconsequential to the final model solution.

Because of the discrete nature of default choices, the government's problem is not differentiable. Therefore, I construct a grid for both state and choice variables. At each state, I search over the government's choice grid to calculate its optimal strategy. While relatively inefficient, this method does guarantee the existence of a solution. From a terminal period, I iterate backward until the decision rule (on the ergodic set) converges. Having solved the model, I calibrate my model to the Mexican economy.

[29] As in LW, I choose to analyze only equilibria with degenerate monetary distributions. This assumption, however, is not trivial and requires that DM sellers always find it optimal to spend the entirety of their DM earnings in the CM market. If this condition does not hold, sellers and buyers will end the period holding different amounts of money. Therefore, a degenerate monetary distribution must have $\phi_t \mu_t + borr_t \leq X_t^{opt}(A_t)$, where $borr_t$ describes the net borrowing revenues in the current period. Further details and explanations are included in the appendix.

## 3.1 Calibration

This model is calibrated to match key observed moments in the Mexican economy, a country with relatively frequent debt crises in recent history.[30] As measured by Reinhart and Rogoff (2009), the past two centuries have seen eight episodes of default by the Mexican government. Thus, Mexico with its frequent history of default is a useful candidate for exploring the determinants and characteristics of sovereign default.

All data used in this paper are annual and run from 1980 to 2010. The World Bank Database is used for data on output, labor supply, and interest rates. The Reinhart and Rogoff dataset is used for identifying crisis episodes, as well as inflation and debt-to-GDP data. Reinhart and Rogoff define a default episode as any failure to meet interest or principal payments on public debt obligations. The data for the currency composition of sovereign debt are taken from multiple sources.[31]

In my model, the stochastic labor productivity process is estimated using Mexican GDP and labor force data. Labor productivity $A_t$ is defined as the ratio of GDP to employed labor force ratio. I assume that labor productivity follows an AR(1) process:

$$log(A_t) = (1-\rho)log(\bar{A_{t-1}}) + \rho log(A_{t-1}) + \epsilon_t$$

[30] Growth in petroleum revenue and easy international credit enabled the Mexican government to borrow vast sums during the late 1970s and early 1980s. Sovereign bond issues rose as the government engaged in a variety of public investment projects. Throughout 1980 and 1981, global interest rates rose while petroleum prices fell. These concurrent events severely eroded the government's fiscal position, quickly pushing the Mexican government into crisis. Unable to roll over its sovereign debt at reasonable interest rates, in August of 1982 the Mexican government suspended interest payments on its bond issues for 90 days. After two months, interest payments resumed when the government received a $4.5 billion rescue loan from the United States and the International Monetary Fund. Then again, in 1994, Mexico found itself in a similar situation when it required an even larger rescue package to prevent default. It is very difficult to identify the exact nature of the shock that led to each episode. Nonetheless, in my paper I assert that productivity shocks contribute to default decisions.

[31] Data on the currency composition of sovereign debt only date back to 1980 at the earliest. The sources for this data include the Bank of International Settlement sovereign debt tables, Cowan et al. (2006), Inter-American Development Bank database, and the Jeanne and Guscina International Monetary Fund dataset.

where

$$\epsilon_t \sim N(0, \sigma_A^2).$$

I detrend both the output and labor data by taking the *log* of the output time series filtered with a linear trend. I find the persistence parameter $\rho = 0.532$ and the variance of detrended labor productivity $\sigma_A^2 = 0.013$. Using a quadrature-based technique, I discretize the innovation process into a 16-state Markov chain.

Sturzenegger and Zettlemeyer (2008) calculate debt haircuts for debt restructuring episodes between 1998 and 2005. They find that the haircuts vary from 18 percent (Ecuador, 2000) to 81 percent (Argentina, 2005). Additionally, Benjamin and Wright (2009) find that creditor losses in sovereign debt negotiations average about 40 percent, consistent with the majority of the haircut sizes in Sturzenegger and Zettlemeyer (2008). I choose to set the debt haircut parameter, $\gamma$, equal to 0.60, consistent with the estimated haircuts in both papers.

Preferences are assumed to take the following form:

$$u(x) = \frac{x^{1-\sigma_{c_1}}}{1-\sigma_{c_1}} \quad c\,(h) = h$$

$$U(x) = B\frac{X^{1-\sigma_{c_2}}}{1-\sigma_{c_2}} \quad C\,(H) = H.$$

Using the aforementioned data series, I calibrate my model to the Mexican economy. I attempt to match the following data moments: debt-to-GDP ratio, monetary velocity, average retail markup, and default frequency using the following five parameters: $\{\sigma_m, \sigma_{c_1}, \theta, \beta, \tau\}$. I use the Reinhart and Rogoff data for average debt-to-GDP ratio (0.39), inflation (26.78 %), and default frequency (4.91 percent chance per year). The average monetary velocity for Mexico from 1980 through 2000 is 11.82 as in Duczynski (2004). As in Aruoba et al. (2011), I use 1.3 as my target retail markup over cost. I set $\sigma_{c_2} = 3$ and set the average of the productivity process $\bar{A} = 0.3$. I restrict the DM utility parameter $\sigma_{c_1} \in (0, 1)$.[32]

[32] Assuming that $\sigma_{c_1} \in (0, 1)$ is done to ensure $u(0) = 0$. Similar models in papers such as LW and Martin (2013) assume $u(x) = \frac{(x+\kappa)^{1-\sigma_{c_1}}}{1-\sigma_{c_1}} - \frac{\kappa^{1-\sigma_{c_1}}}{1-\sigma_{c_1}}$ to avoid requiring $\sigma_{c_1} \in (0, 1)$. However, inclusion of

The model moments I use to match the data are defined as follows: GDP is defined in terms of CM market goods and is equal to $Y_t \equiv \frac{\mu_t \phi_t}{A_t} + X^{opt}(A_t)$. Monetary velocity is generally defined as nominal expenditures divided by the total money supply. In my model, velocity can be written as $\frac{\phi_t Y_t}{M_t}$. Gross CM inflation is equal to $\frac{\phi_t}{\phi_{t+1}}$, and the retail markup can be expressed as $\frac{\phi_t \mu_t}{x_t}$, the ratio of the real CM revenues to the seller to the cost of DM production in terms of CM goods. The default frequency is defined as the total number of default events divided by the total number of periods. Lastly, incoming period debt divided by GDP, $Y_t$, I call the debt-to-GDP ratio.

To calibrate, I first solve the model using a given parameterization. Then, I generate 1 million productivity realizations using the specified process for productivity. Employing the equilibrium policy function, I can trace out the evolution of the economy given the simulated sequence of productivity shocks and calculate implied model moments.[33] Upon calculating model moments, I minimize the sum of absolute deviations between model moments and target moments over the set of possible parameterizations.

# 4 Results

I present results from my model in four parts. First, I isolate the role of monetary policy by solving the model without any forms of debt. Then, I proceed to present the results of my model when the government has access to only dollar-denominated debt. Then, I present the results when the government has acces to only peso-denominated bonds. Lastly, I allow the government to borrow using both local and foreign currencies. I calibrate this full model to the aforementioned data moments, describe optimal policy, and present an analysis of a default event.

this $\kappa$ variable has proven very distortionary in my work. For values of $\kappa \approx 0$, computing utility precisely becomes difficult because of limited computational precision. Note that the second term in the utility function approaches infinity as $\kappa$ approaches 0. When, alternatively, $\kappa$ is large, utility values can be precisely computed; however, the marginal utility of consumption becomes distorted. In fact, for large enough values of $\kappa$ it will be the case that $u'(0) < c'(0)$. Therefore, to avoid either of these issues, I restrict $\sigma_{c_1} \in (0, 1)$.

[33] I throw out the first 1,000 observations when presenting simulation results.

## 4.1 No Debt

When the government is unable to issue debt, the policy choice set is reduced to only the growth rate of the money supply, $g_t$. Monetary policy is administered to maximize total surplus in the economy by minimizing monetary frictions. Total surplus is equal to the sum of centralized market utility $U(X_t) - C(\frac{X_t}{A_t})$ and decentralized market utility $\sigma_m \left(u(x_t) - c(\frac{x_t}{A_t})\right)$. The centralized market, being a simple Walrasian market, always yields the optimal production level $X^{opt}(A_t)$. However, matching and bargaining frictions create inefficiencies in a decentralized market. The government will adjust monetary policy to reduce these frictions as much as possible.

To do so, the sovereign will optimally choose the growth rate of money that yields the highest level of decentralized market production, thereby reducing the decentralized market's hold-up problem. Through monetary policy, the government can essentially pick the Nash bargaining equilibrium that yields the largest DM production outcome possible, $x_t = \bar{x}_t$.

**Proposition 1 (Optimal Money Growth Without Nominal Debt)** *When the government does not have access to nominal debt, it will choose to increase the money supply at a rate of $g_t = \bar{g}(S_t)$. See the appendix for proof.*

In Figure 3, I present the sovereign's optimal choice of money growth for each possible productivity level. In the top panel are three money growth thresholds. First, $g^{min}$ displays the lowest possible rate of money growth consistent with a degenerate monetary distribution.[34] In addition, $g^{opt}$ shows the upper bound of the money growth rate that could achieve the first best solution such that DM production is maximized at $x_t = x^{opt}(A_t)$, if that were possible. Lastly, $\bar{g}$ describes the growth rate of money that yields an equilibrium with the highest feasible production level $\bar{x}$. The solid line $g$ displays the government's chosen policy. As shown, the optimal choice for the government is to grow the money supply at the highest

[34] For sufficiently low rates of monetary growth, the DM transfer will always be too large to be sold back to DM consumers in the CM. Under the model's assumptions, DM sellers would always leave the CM market with more money than their consumer counterparts.

rate that still guarantees the constrained optimal equilibrium. The second panel displays the DM production levels implied by each of the thresholds and the actual choice of growth rate for the money supply.

My model predicts both money growth and prices to be countercyclical. By proposition 1, an optimal money growth rule without local currency debt is $g_t = \bar{g}_t$ where $\bar{g}_t \equiv \frac{\hat{\phi}_t}{A_t z(\bar{x}_t, A_t)}$. Using this optimal currency rule and equation (7), I can solve for the equilibrium normalized value of money in the CM:

$$\frac{M_{t+1}}{p_{c,t}} = \hat{\phi}_t = \beta A_t E_t \left[ z(\bar{x}_{t+1}, A_{t+1}) | A_t \right].$$

Then, it must be the case that:

$$\bar{g}_t = \frac{\beta E_t \left[ z(\bar{x}_{t+1}, A_{t+1}) A_{t+1} | A_t \right]}{z(\bar{x}_t, A_t) A_t}.$$

These two conditions imply countercyclicality of prices and money growth. It can be shown that $z_A(\bar{x}_t, A_t) > 0$ for all relevant parameterizations.[35] Given that productivity shocks follow an AR(1) process, both the equilibrium normalized price level and the chosen rate of money growth (under the assumed monetary policy rule) will fall as productivity rises. The purchasing power, and thus the value, of money rises with productivity. This explains why prices and productivity are negatively related.

However, the intuition behind countercyclical money growth is slightly more subtle. When productivity is high, the price of decentralized market goods is relatively low and therefore the value of money is also relatively low. Because the productivity process displays mean reversion, agents expect lower productivity, higher prices and a higher value of money in the future. To entice people to spend their money today rather than waiting to spend it when its value is higher, the government must increase its value today by reducing the supply. Therefore, to sustain the highest feasible production level $\bar{x}$ when productivity is high,

[35] Although this cannot be proven analytically, it is checked numerically.

the government sets a low rate of money growth (or higher rate of monetary contraction). Alternatively, when productivity is low, the government will allow the money supply to grow at a higher rate to ensure that money does not become overly valuable.

Kaminsky et al. (2004) find that while advanced countries typically employ countercyclical monetary policy, monetary policies in emerging market economies tend to be procyclical. Calderon et al. (2004) suggest that government credibility (as proxied by the sovereign risk spread) is highly correlated with countercyclical policies in EM countries. When sovereign risk spreads rise, monetary policies tend to move procyclically. In my model, there is no notion of regime credibility because, in a given period, the government behaves as a separate entity from the preceeding one. Therefore, it will be difficult for a model such as this to replicate procyclical monetary policies. However, I will return to these empirical observations later in this paper as I add debt to the model to see how sovereign risk spreads and money growth rates interact.

In the final panel, expected inflation $E_t\left[\frac{\phi_t}{\phi_{t+1}}\right]$ rises with productivity. When productivity is low, the price of centralized goods is relatively higher because those goods are more costly to produce. Because agents expect productivity to rise during period when productivity is low, they expect prices to fall in the future, leading to deflation (or lower inflation). As productivity rises, agents expect higher future prices, leading to higher expected inflation.

Money growth and expected inflation tend to move in opposite directions. Money growth acts to dampen this inflationary trend and ensure that money is sufficiently valuable even at high productivity levels. This policy serves to achieve the highest possible level of social surplus. Monetary policy smooths these pricing dynamics and ensures economic efficiency.

Again, it is clear that assuming a degenerate distribution of money holdings is not trivial. The money growth limit that ensures a degenerate monetary distribution is higher than the level necessary to achieve the socially optimal outcome. Therefore, not only is $x^{opt}$ unattainable as a Nash Bargaining equilibrium, but it is also infeasible as an equilibrium with a degenerate distribution. In addition, as productivity increases, the degenerate distribution threshold will eventually bind. This occurs when $z(\bar{x}_t, A_t) \geq X^{opt}(A_t)$ (real value of the transfer exceeds the point at which $U'(X) = C'(\frac{X}{A})$) and indicates that optimal monetary policy for the government no longer implies a degenerate distribution.

### 4.2 Dollar Debt

Before describing the dynamics of a model with both peso and dollar debt, I will next explore the characteristics of my model when the sovereign can issue only foreign currency debt. As in Arellano (2008), there exists a threshold level of debt $b^*_{def}(A)$ for every productivity draw. This threshold describes the largest stock of incoming debt that a government would be willing to repay without defaulting. In other words, the government will honor all debt obligations $b^*_t < b^*_{def}(A)$ and default on all debt levels $b^*_t \geq b^*_{def}(A)$ when it receives a productivity draw $A$.

**Proposition 2 (Default Thresholds)** *For each productivity level A, if the government chooses to default on any $b^* \in [0, \infty)$ then $\exists$ $b^*_{def}(A)$ such that the government chooses to default for $b^* \geq b^*_{def}(A)$ and chooses not to default $b^* < b^*_{def}(A)$. See appendix for proof.*

Figure 4 describes how this threshold (or debt limit) varies by productivity innovation in a model with only dollar debt. The borrowing threshold $b^*_{def}(A)$ decreases in productivity as well as output.[36] Although at first glance this might seem counterintuitive, this result makes sense in the context of consumption smoothing. The cost of default is two-fold: the direct utility penalty (which is lower when productivity is low) and the cost of repaying the debt less the haircut (which is higher when productivity is low). When productivity is low, the benefit of borrowing is the highest. Therefore, to enable the government to borrow when productivity is low, the government chooses strategically to repay its debt instead of defaulting. If productivity falls further, it will still have the ability to borrow more. However, if productivity improves, the government will choose to default on its debt

[36] This finding echoes the results of Arellano (2008).

while taking advantage of the productivity boost. When labor costs are high relative to expected future costs, the sovereign needs to borrow much more than when costs are lower. To protect its ability to borrow, the government borrows heavily from abroad to ease its citizens' labor burden. During these events, default probabilities and borrowing costs rise. When productivity rises, the government is saddled with high debt but can more easily repay that debt, especially after defaulting.

To further examine this dynamic, the policy function for a government with a high productivity draw is presented in Figure 5. In these states, the government chooses to borrow up to a level where it assumes no default risk. The combination of low labor costs and relatively high default costs implies that the government's borrowing behavior remains conservative when productivity is high. During those periods, lenders are willing to lend more but the government is unwilling to take on more debt because the government is protecting its fiscal flexibility in future periods when productivity could be lower.

When the sovereign receives a low productivity draw, the desire to defray high labor costs leads to the assumption of more debt and positive default risk. Just as in the high productivity case, when the government enters the period with low debt levels, it proceeds to borrow funds with no associated default risk. However, as shown in Figure 6, if the sovereign enters the period with $b_t$ at or below the risk-free amount, it chooses to borrow a level higher than its default threshold. In fact, the amount borrowed will be high enough to exceed the default thresholds in all states of the world. Although that scenario may seem counterfactual, recall that this model contains only one-period bonds and therefore is unable to match the dynamics of debt and default that occur when countries can borrow at longer maturities.

Borrowing at a level universally above the default threshold guarantees default in the following period. Because default in the following period is certain, the bond price will reflect the fact that lenders expect to recover only the principal less the haircut. Therefore, in those circumstances, the bond price will drop to $\frac{\gamma}{1+r^f}$, the risk-free rate less the default haircut.

Given the assumed process for $A_t$, when productivity is below average today there is a high probability that productivity will rise in the next period. If productivity rises following a jump in the debt level, the sovereign reaps the benefit of lower labor costs but will not want to roll over its high level of debt. Even though the default penalty is relatively more burdensome (in terms of utility) in high productivity states, the labor costs of repaying the debt minus the haircut are lower than in the previous period. Thus, in the next period, if productivity rises as expected, the government will default, absorb the penalty, and return to a low level of debt. This strategy implies that default events do not occur in the first period of a large negative economic shock but in the periods that follow that negative shock, consistent with the findings of Tomz and Wright (2007).

When the government enters a period with no debt, it will always borrow such that it bears no default risk. If, following this initial period of borrowing, productivity is low, the government proceeds to borrow such that it will always default in the next period. If, however, productivity is high, the government chooses to roll over that same amount of debt. When the government enters the period with debt beyond the risk-free amount, the government always chooses to default in the current period. The expected return on this debt—in stark contrast to the return on peso debt as will be seen—does not vary by productivity draw.

## 4.3 Peso Debt

When the government has access only to locally denominated bonds, it has to balance its two monetary policy motives: promoting the efficiency of the DM and reducing the real burden of nominal debt. Reconciling these two motives can be seen in the determination of the domestic price function. In equilibrium, the ex-ante real return on local currency bonds (as is also the case with foreign currency bonds) is equal to the risk-free rate $r^f$. Therefore, the government *cannot* adjust the expected real return on these bonds.[37] Monetary policies effectively manipulate how the expected real cost of local currency debt is spread over future states. A consumption-smoothing sovereign with local currency debt would choose a countercyclical price function. A pricing function that falls with productivity induces a negative relationship between productivity and the real burden of nominal debt. When the pricing function is countercyclical, in low productivity periods the government would have to send fewer goods abroad to pay off the same amount of nominal debt than it would in high productivity periods.

In equilibrium, I find that the government prefers to erode debt through inflation rather than through explicit default.[38] Unlike in the model with only dollar-denominated debt, money growth choices are influenced by the level of debt. Figure 7 shows how money growth rates depend on the ratio of peso-denominated debt to the total money supply (the relevant state variable for nominal debt). In this diagram, optimal money growth rates are increasing in the amount of incoming local currency debt. When the government has no debt, the best choice for the government is still $g_t = \bar{g}_t$. However, as debt rises, so too does the chosen rate of money growth. The three lines indicate how much debt the government would need to have to find it optimal to choose 10 percent, 50 percent, and 100 percent money growth.

Policy functions in this model do not vary significantly in shape by productivity. Figure 8 presents a policy function in this model when the government receives an average productivity draw. The panels in this figure describe the government's optimal borrowing and money growth choices given its incoming debt level. In addition, the figure shows the money growth rates, consumption levels and expected inflation rates implied by the government's choices.

[37] This stems from a no arbitrage condition and also holds true for dollar-denominated bonds. The price of borrowing an amount of bonds is set such that in expectation, the return is equal to the risk-free rate. With respect to dollar-denominated borrowing, the price today reflects the probability that the government will default in the following period. With respect to peso-denominated borrowing, the price today reflects not only the probability of default but also the probability that price changes will affect the real return on those bonds. However, ex-ante, the expected real return on both types of bonds is equal to the risk-free rate.

[38] Peso default can be induced, but these parameterizations must have exceptionally low default costs. The values of $\tau$ that yield peso default episodes imply a default penalty of less than 0.01 percent of CM production.

As can be seen, higher incoming debt levels are associated with higher optimal money growth rates, higher inflation rates, higher nominal borrowing costs, and lower consumption levels.

The real burden of peso debt will be larger in states where the price level is lower and the value of money is higher. Figure 9 illustrates how the real return on peso-denominated debt varies depending on the future productivity draw. In this figure, the sovereign borrows when productivity is low. The vertical line indicates the productivity level at which the government borrows today. The horizontal lines show the difference between the real bond revenues today and the expected value of repayment in the next period. Note that the distance between these two lines represents the expected real rate of return. The diagonal line in this diagram indicates how the next period's borrowing cost varies by productivity realization. Because the price level and productivity are *negatively* correlated in equilibrium, negative productivity shocks serve to lower the burden of domestic debt. Alternatively, when productivity improves, the real cost of repayment rises. In this example, when productivity is below average, if the sovereign receieves the same productivity draw during the next period, the real return on its debt would actually be negative.

The dynamics of domestic prices create a unique and important role for local currency debt. The government can borrow in pesos when productivity is low. If productivity remains low, the cost of that debt is relatively low. However, if productivity rises, the real burden of that debt rises and offsets the gains from its positive shock. Peso-denominated debt allows the government to hedge against future shocks.[39] Figure 10 presents the payoff function when productivity is high. Here, peso-denominated debt provides insurance against a fall in productivity. Unlike the low productivity example, the realized cost of borrowing if the sovereign receives the same high productivity shock would be well beyond the expected real borrowing cost.

Figure 11 illustrates how the government manipulates the state-contingent payoff on local currency bonds. In this diagram, I compare price functions corresponding to different

[39]This result supports the argument put forth in Bohn (1988).

levels of incoming local currency debt. The lowest curve describes the price function across productivity draws when the government enters the period without any debt. As expected, without any debt the government sets $g_t = \bar{g}_t$ and prices are determined accordingly. The more concave the pricing function, the more the government is willing to inflate away debt at low productivity levels. The other two curves describe pricing functions when peso debt is at a low level and then when peso debt is high. When local currency debt is high, the government is far more willing to manipulate prices to reduce debt burdens at low productivity levels. Essentially, higher levels of debt increase the incentive to engage in strategic inflation. In doing so, the government resigns itself to larger real payoffs when productivity is high. This diagram captures the dynamics behind strategic inflation.

The value to the sovereign of issuing peso-denominated debt is constrained by the government's inability to commit to future policies and its sensitivity to inflation. The less costly inflation is, the more likely the future government is to employ strategic inflation to erode nominal obligations. Costly inflation ties the hands of future governments, allowing for a higher sustainable amount of peso-denominated debt. When the costs of inflation are low, however, the threat of strategic inflation dampens the hedging capabilities of local debt issuance.

## 4.4 Dollar and Peso Debt

Before calibration, it is prudent to gain insight into how key parameters affect model dynamics. I begin my analysis by considering how the relevant calibrated variables affect key model moments. I simulate my model under a variety of parameterizations, calculate implied model moments from each simulation, and estimate the partial derivatives of relevant model moments with respect to the set of calibration parameters. Because model dynamics are nonmonotonic, this provides a useful look into how model parameters generally affect the model's equilibrium. Table 1 presents these correlations. The model moments presented include the gross inflation rate, monetary velocity, the DM markup, money growth, the price/money supply ratio, the relative size of CM production, total DM production, default, the percentage of debt denominated in pesos, as well as the debt-to-GDP ratio.

Some clear and unsurprising relationships emerge from this exercise. As expected, $\tau$, the cost of default, is positively correlated with the debt-to-GDP ratio. However, I find that this penalty is negatively correlated with the frequency of default. Those relationships indicate that as default becomes costlier, governments are more reluctant (although only weakly) to choose to default at any debt level. Reducing the set of states in which the sovereign chooses to default expands the risk-free set of borrowing choices, reduces borrowing costs, and yields equilibria with higher sustainable debt levels. The buyer's bargaining weight $\theta$ is strongly negatively correlated with the decentralized good's markup over cost. Naturally, as the seller's bargaining power falls, so too do the margins that producers earn on production. In addition, as expected, the CM utility weight $B$ is positively correlated with the percentage of economic production in the centralized market.

The determinants of the currency composition of sovereign debt are of particular interest. Higher levels of locally denominated borrowing are correlated with low default costs and higher weight on CM utility. First, consider the relationship between debt denomination and the cost of default. When default costs are low, relatively low levels of foreign-denominated debt can be supported in equilibrium. As shown in the two previous models, governments will default in equilibrium on dollar debt but will choose erosion through strategic inflation rather than default when borrowing in local currency. Thus, default costs should have asymmetric effects on borrowing capacity for each type of debt. This result is slightly counterintuitive, as higher default costs allow the government to sustain higher levels of borrowing. Rising default costs here increase the relative riskiness of local currency debt relative to foreign currency debt, decreasing the percentage of local currency borrowing to total real borrowing.

The relationship between domestic currency borrowing and the CM utility weight is derived from the costs of inflation. The more that agents care about CM consumption, the higher the equilibrium level of peso-denominated debt. In this model, the costs of inflation are realized in the DM. As the weight on centralized market utility increases, agents will be less sensitive to lost DM consumption. When agents are less sensitive to inflation, the government will be more willing to use monetary policy to manipulate the peso-debt payoff function. The more tolerant the agents are of strategic inflation, the higher the sustainable amount of local currency debt in equilibrium. Therefore, any parameterization that allows agents to be more tolerant of inflation should increase the share of peso-denominated debt in the sovereign's debt portfolio. For this reason, the relative share of nominal debt increases as the utility weight on DM utility falls.

Through the calibration routine, I obtain a set of parameter estimates that best matches model moments to data moments. These parameters and their implied model moments are presented in Table 2. My calibration is able to replicate some of the Mexican data moments. The debt-to-GDP ratio and the interest rate on foreign currency debt are both very close to their data counterparts. However, the model has difficulty matching both the default probability and the average rate of inflation. New Monetarist models typically generate optimal monetary policies that follow the Friedman rule.[40] In this model, nominal debt provides a disincentive to allow deflation, yet this motive is not nearly powerful enough to yield positive rates of money growth. Therefore, while I will be unable to replicate realistic inflation rates, I will be able to describe how elements of my model alter the dynamics of money growth and inflation.

The parameters listed in Table 2 yield the closest attainable matches to the data moments. The parameter $\tau$ has important implications for default and borrowing levels. The calibrated value $\tau = 1.02$ is more costly when productivity is high. When productivity is low, this penalty is equivalent to an 8.4 percent decrease in contemporaneous CM consumption. When productivity is high, this cost rises to 9.8 percent of CM consumption.

[40]In this model with stochastic productivity, the optimal inflation rate is $\beta E_t \left[\frac{A_{t+1}}{A_t} | A_t\right]$. Aruoba and Chugh (2010) and Gomis-Porqueras and Peralta-Alva (2010) all modify the LW framework and generate equilibria wherein the Friedman rule does not always hold and positive nominal interest rates may be optimal.

As shown in the final row of Table 2, my model significantly underpredicts the average ratio of local currency to foreign currency debt. One reason for this result is the calibrated value of $B$, the weight on CM utility. The value for $B$ needed to fit the debt-to-GDP ratio is especially high. That value implies a low relative weight on DM utility. As discussed previously, a high value for $B$ causes the sovereign to place less emphasis on DM efficiency, yielding a lower sensitivity to inflation and a low sustainable amount of local currency debt. In addition, as will be discussed later, constant foreign prices understate the relative riskiness of foreign currency debt, leading to a larger percentage of foreign currency equilibrium borrowing than would arise if foreign prices fluctuated.

In Table 3, I present correlations among important variables in my simulated economy. For comparison, in Table 4, I calculate corresponding moments from Mexican economic data. In general, peso borrowing in my model is not strongly correlated with model moments. Albeit weakly, I can successfully match the negative correlation between local currency borrowing and inflation. In addition, the model proves successful in matching most data correlations with the interest rate spread. This indicates that although it is unclear whether I can successfully match the dynamics of the monetary economy, my model succeeds at matching foreign currency default and borrowing dynamics.

## 4.5 Model Dynamics

Sovereign borrowing decisions vary with productivity realizations. To illustrate how exactly productivity shocks influence borrowing behaviors, I simulate the model economy when the country receives, by chance, the same productivity realization every period for each level of productivity. In this experiment, all agents expect productivity to revert to the average but are always wrong. This exercise helps isolate the ways in which government strategies vary by productivity. For each productivity draw, I simulate the economy for one million periods, throwing out the first 1,000 observations. In Table 5, I present the implied model moments for each simulation.

The percentage of debt denominated in the local currency declines substantially as productivity increases from the lowest productivity draw. The reason for this trend is two-fold. First, because prices decline with productivity, a government with a low productivity draw can smooth consumption through local debt issues more effectively than with foreign currency debt. An increase in future productivity brings both lower labor costs, but a higher burden of debt. If, however, productivity remains low and prices stay high, labor costs are high but the burden of peso-denominated debt is relatively more manageable.

In addition, sovereign default in my model occurs in equilibrium but only with respect to foreign-denominated debt. In periods of default, the sovereign is temporarily excluded from borrowing using dollar-denominated bonds. In these periods, the government is forced to issue domestically denominated debt if it wishes to borrow at all. The productivity levels at which domestically denominated borrowing represents a large share of total borrowing coincide with the highest default frequencies. This result is not without an empirical basis. Countries that default on foreign creditors are often forced to borrow in alternative debt markets before they gain re-entry to external markets.[41]

The proportion of debt denominated in pesos also increases slightly as productivity rises above its average level. This result is driven by the hedging characteristic of local debt. As discussed earlier, when productivity is high, peso debt softens the blow from a fall in domestic productivity as the higher cost of labor is tempered by a lower burden of debt. The government, therefore, has the largest incentive to issue local currency debt when the gains from its hedging motive are the highest. The benefits from hedging are largest when the expected productivity changes are the largest. Due to the assumed productivity process, this occurs when productivity is either extremely high or extremely low. This explains the eventual rise in the percentage of peso debt that occurs as productivity rises.

[41] In the years since Argentina's default in 2002, the country relied upon loans from sources such as the World Bank, Inter-American Development Bank, the Venezuelan government and a number of private banks for funding. Borensztein et al. (2006) assert that for many countries, crisis episodes served to initiate a country's local currency sovereign bond market. They point to Mexico and Uruguay where crisis events in the 1980s prevented them from borrowing in international markets and instead forced their governments to issue bonds domestically.

In low productivity simulations, governments tend to issue more debt relative to GDP, default more often and pay foreign creditors significantly more than when governments receive high productivity draws. In addition, money growth and expected inflation tend to decrease in productivity.[42] Bond prices on foreign-denominated debt rise with productivity as default probabilities fall.

Furthermore, as was suggested in the models with only dollar borrowing and only peso borrowing, changes in monetary policy are driven primarily by the amount of peso debt. This result is shown in Figure 12, which shows the optimal money growth rate as a function of both incoming dollar debt and incoming peso debt. Money growth increases primarily along the peso debt axis and slightly along the dollar debt axis. The positive correlation between money growth and local currency debt is unsurprising. However, contrary to the findings in the model with only dollar debt, dollar borrowing does indeed affect the optimal growth rate of money. This result is interesting because it shows clearly that foreign currency debt can affect equilibrium monetary policy decisions and that excessive foreign currency debt may contribute to high rates of inflation. Here we see clearly that higher amounts of incoming foreign currency debt, when the government has local currency obligations, provide a stronger incentive for strategic inflation.

## 4.6 Welfare Analysis

I now explore the welfare implications of having access to both foreign and domestic currency bonds. In my first exercise, I simulate eight versions of my model. First, I run the model without sovereign debt, with only peso debt, only dollar debt, and then both types of debt. In these four versions of the model, the government has complete discretion

[42] One exception occurs at the productivity level $A = 0.27$. Money growth is significantly higher at this productivity level, which represents a barrier between productivity draws where implied default probabilities tend to be positive and those where default occurs with trivial probabilities. At this level of productivity, the incentive to reduce debt burdens is strong but not quite strong enough to incentivize default. Here, the government will prefer debt reductions through inflation rather than face the large cost of explicit default. At this productivity level, strategic inflation is the most useful—when the costs of default are just high enough to be prohibitive, but the incentive to reduce debt obligations is still strong.

over monetary policy. Next, I run the same four iterations of my model, except under the assumption that the government is forced to commit to a monetary policy rule. This "optimal policy rule" limits the sovereign to choosing money growth such that $g_t = \bar{g}$ each period, maximizing feasible decentralized market production. This exercise tries to ascertain whether monetary policy discretion can be detrimental to the sovereign.

The results of these simulations are described in Table 6. In this table, I measure the differences in welfare between each iteration of my model and a benchmark model in terms of the implied difference in average CM consumption during each period. The benchmark model used is the full-commitment no-debt case. Access to dollar debt improves welfare over the benchmark case significantly, by 6.75 percent of average CM consumption. Gaining the use of only peso-denominated borrowing has a far weaker benefit, increasing welfare by merely 0.03 percent each period. Two elements contribute to these large differences in welfare between each of these debt instruments. First, because local currency debt is state-contingent, its payoff function will be more variable than the payoff on dollar bonds. Even though this debt can be used to hedge against productivity shocks, the variance of its payoff functions makes it relatively less appealing for a consumption-smoothing sovereign. Thus, the government protects against large swings in the cost of issuing local currency debt by issuing very little relative to foreign currency debt.

The assumption of a constant foreign price level overstates the stability of foreign prices as well as the relative volatility of local prices. This is the main reason why access to local debt has such a low utility benefit and why I am unable to match observed local currency debt percentages. It would stand to reason that making the dollar bond payoff more volatile (through stochastic shocks to $\phi_t^*$) would serve to increase the relative value of local debt and shift the portfolio composition toward local currency debt.

Welfare is strictly improved by the ability to issue both peso-denominated and dollar-denominated debt. In addition, when the sovereign has access to both peso and dollar debt, I find that both types of debt are issued in equilibrium. Building on the results from Arellano and Heathcote (2010), my model demonstrates that a government that borrows in both foreign and local currency can borrow even more than a dollarized regime. However, as is shown in the fourth line of Table 6, borrowing using both debt instruments yields higher equilibrium inflation than in the dollar debt or no-debt regime.

In this model, the sovereign's inability to commit to future monetary policy limits its ability to issue local currency debt. In the second set of simulations in Table 6, monetary policy is restricted to $g_t = \bar{g}$ every period. Under commitment, sustainable peso-denominated debt increases and inflation falls. This finding suggests that monetary policymakers with discretion may be unable to prevent themselves from printing too much money and eroding away nominal debt. When governments lack credibility to commit to low rates of money growth, it will be nearly impossible to enact first-best policies. Thus, restrictions on the choice set of each government will change equilibrium decisions for both current and future governments alike. This result reflects the findings of Calvo (1978), Barro and Gordon (1983), and Lohmann (1992) who describe how in the presence of time-consistency issues, credible monetary policy commitments can improve economic outcomes. Here, inability to commit to low rates of money growth reduces the local currency borrowing capabilities of the government. Once the hands of the policymakers are tied and strategic inflation is no longer possible, local currency borrowing rates rise.

In Figure 13, I show the welfare cost of discretionary monetary policy in this model of sovereign debt. I simulate the model with both types of debt when money growth choices are limited to different-sized windows around $g_t = \bar{g}$. In the first simulation, the government commits to printing money at a rate equal to $\bar{g}$ during each period. The subsequent examples show what happens as money growth is restricted to windows of $\pm 5$ percent, $\pm 10$ percent, and $\pm 25$ percent around the money growth rate $\bar{g}$. This figure measures differences in welfare at each possible productivity level between the benchmark model (full monetary policy discretion) and the different monetary policy regimes, when the sovereign enters the period with no debt. Two clear trends appear in this graph. First, the value of monetary policy commitment declines with productivity. The hedging capability of peso-denominated debt is highest when expected productivity changes are largest. When productivity is especially low, the demand for borrowing is highest and the value of local currency debt is at its highest. Therefore, the largest welfare gains from preventing strategic inflation arise at low levels of productivity. In addition, the graph illustrates the value of monetary policy commitment in a model with nominal debt. As the money growth window expands, the welfare gains over the baseline model shrink. In Table 7, I present descriptive model statistics for each iteration of the model. As the money growth window expands, total borrowing and local currency borrowing declines, welfare declines, monetary velocity declines, and average inflation increases.

The value of monetary policy commitment depends inherently on the rate of money growth that the government commits to. Commitments to low rates of money growth yield better results than commitments to high levels of money growth for this economy. This idea is illustrated in Table 8. In this table, I alter the rate of money growth to which the government commits. Note that welfare in this table is not maximized by $g_t = \bar{g}$ but by a commitment to an even lower rate of money growth $g_t = \bar{g} - 20$ percent. Adherence to money growth rates below $\bar{g}$ does not influence the efficiency of the DM, however, it does serve to lower the variance of prices. This result shows another difference in the characteristics of local and foreign currency debt in my model. Decreasing the variance of domestic prices dampens the volatility of local debt payoffs, making local currency debt more appealing as a hedging tool. Therefore, a commitment to a low rate of money growth decreases price volatility and leads to more local currency debt issuance. Under full discretion, the government cannot commit to these welfare-improving policies.

# 5 Conclusion

In this paper, I have developed a sovereign debt model that endogenizes monetary and fiscal policy choices in emerging economies to gain insights into the government's decisions about debt denomination. Monetary and fiscal policy choices are co-determined. My model replicates the inherent tension between an emerging market government's ex-ante desire to borrow in the domestic currency and its ex-post desire to employ strategic inflation to reduce the real debt burden. Monetary policy choices alter the borrowing capabilities of the sovereign. I show that, in an emerging economy, a credible commitment to a monetary policy rule serves to increase sustainable local borrowing levels, total debt levels, and welfare. The optimal currency composition of debt is a function of a government's monetary policy credibility and sensitivity to inflation. In addition, I calibrate my model to the Mexican economy. Model simulations yield equilibrium default on foreign-denominated debt, but the government described in the model resorts to strategic inflation to reduce local currency debt.

My model lends itself to a variety of alterations that may improve its ability to match empirical facts. First and foremost, adjusting the bargaining mechanism in the DM may allow for better matching of inflation dynamics. Alternatives, such as egalitarian bargaining, reduce the cost of inflation and will make it less costly for the government to manipulate the payoff function for peso-denominated debt (leading to higher sustained levels in equilibrium). In addition, all else being equal, a government's ability to inflate away its nominal debt is invariably determined by the duration to maturity of such debt. Therefore, the next logical extension to this model would be to add long-term nominal debt. Using a Hatchondo and Martinez (2009) bond framework would allow me to incorporate long-term local currency debt. This would provide an avenue to study debt dilution questions as well as the interactions between monetary policy and debt maturity length.

Lastly, it would be useful to relax the assumption of stable foreign prices. Adding uncertainty to the value of the foreign currency would serve to reduce the relative riskiness of local currency debt. The correlation between these domestic and foreign shocks will have extensive implications for monetary and borrowing policy decisions. This addition would enable my model to better match the empirical currency composition of sovereign debt.

The topic of domestic currency sovereign borrowing in emerging economies will remain an important line of research going forward. Understanding the reasons behind government borrowing behaviors necessitates an extensive search for data on debt denomination. While in recent years data on domestic debt have become somewhat more accessible, such data are generally severely lacking. Therefore, this topic has wide room for additional research not only theoretically, but also empirically.

# Appendix

## Solving for the Domestic Economy Equilibrium

Below is presented a comprehensive derivation of the Lagos and Wright monetary economy embedded within the sovereign model in this paper:

### Bargaining Problem Solution

Using the result that the continuation value $W(m_t, S_t; \Omega)$ is linear in money holdings, the decentralized market (DM) bargaining problem can be simplified. This bargaining problem is subject to the following four conditions: a feasible transfer $\mu_t \leq m_t$, positive production $x_t > 0$, consumer optimality $u(x_t) \geq \frac{\phi_t \mu_t}{A_t}$, and producer optimality $c\left(\frac{x_t}{A_t}\right) \leq \frac{\phi_t \mu_t}{A_t}$.

The bargaining problem can be rewritten:

$$\max_{x_t > 0,\, \mu_t \leq m_t} \left[u(x_t) - \frac{\phi_t \mu_t}{A_t}\right]^{\theta} \left[-c\left(\frac{x_t}{A_t}\right) + \frac{\phi_t \mu_t}{A_t}\right]^{1-\theta}.$$

Maximizing with respect to $x$ and $\mu$ yields the following first-order conditions, respectively:

$$x \ : \quad \theta u'(x_t)\left[-c\left(\tfrac{x_t}{A_t}\right) + \tfrac{\phi_t \mu_t}{A_t}\right] - (1-\theta)(\tfrac{1}{A_t})c'\left(\tfrac{x_t}{A_t}\right)\left[u(x_t) - \tfrac{\phi_t \mu_t}{A_t}\right] + \frac{\lambda_t + \xi_t u'(x) - \varphi_t(\frac{1}{A_t})c'\left(\frac{x_t}{A_t}\right)}{\left[u(x_t) - \frac{\phi_t \mu_t}{A_t}\right]^{\theta-1}\left[-c\left(\frac{x_t}{A_t}\right) + \frac{\phi_t \mu_t}{A_t}\right]^{-\theta}} = 0 \tag{8}$$

$$\mu \ : \quad \theta\left[-c\left(\tfrac{x_t}{A_t}\right) + \tfrac{\phi_t \mu_t}{A_t}\right] - (1-\theta)\left[u(x_t) - \tfrac{\phi_t \mu_t}{A_t}\right] + \frac{-\delta_t \frac{A_t}{\phi_t} - \xi_t + \varphi_t}{\left[u(x_t) - \frac{\phi_t \mu_t}{A_t}\right]^{\theta-1}\left[-c\left(\frac{x_t}{A_t}\right) + \frac{\phi_t \mu_t}{A_t}\right]^{-\theta}} = 0. \tag{9}$$

Here $\delta$, $\lambda$, $\xi$ and $\varphi$ are the Lagrange multipliers on the transfer, production, consumer optimality and producer optimality constraints, respectively. The price level $\phi_t$ is taken as given for now.[43] Maximization thus yields two first-order conditions and six unknowns —$x_t$, $\mu_t$ and the four Lagrange multipliers. To help ensure that a monetary equilibrium will be weakly preferred to a nonmonetary equilibrium, it is helpful to impose some basic conditions on the DM utility and cost functions. Therefore, I make the following assumptions: $u(0) = 0$, $u'(0) = \infty$, $u'(x) > 0 \geq u''(x)$, and $c(0) = 0$, $(\frac{1}{A_t})c'(h) \geq 0$ $c''(h) \geq 0$. Because $u'(0) = \infty$, buyers will always demand positive DM consumption, regardless of the price. Then, it must be the case that $\lambda_t = 0$.

First, consider only scenarios in which neither of the optimality conditions holds with equality. In these cases, $\xi_t = 0$ and $\varphi_t = 0$. After describing the bargaining problem solution under this set of assumptions, I will return to evaluate what happens when one or both optimality conditions hold with equality. Here, I can rewrite equation (8) as:

$$\frac{\mu_t \phi_t}{A_t} = z(x_t, A_t) \equiv \frac{\theta u'(x_t) c(\frac{x_t}{A_t}) + (1-\theta)(\frac{1}{A_t})c'(\frac{x_t}{A_t})u(x_t)}{\theta u'(x_t) + (1-\theta)(\frac{1}{A_t})c'(\frac{x_t}{A_t})}. \tag{10}$$

Equation (10) describes how the surplus from the DM match is split between the buyer and seller in equilibrium. This equation defines the relationship between $\mu_t$, the amount of money transferred from seller to buyer, and $x_t$, the amount of goods produced by the seller. I define $z(x_t, A_t)$ as the seller's share of the surplus created from a DM match, given $\theta$, the Nash bargaining parameter. In the Nash bargaining equilibrium, $z(x_t, A_t)$ will be equal to $\dfrac{\mu_t \phi_t}{A_t}$, the utility value of the monetary payment from consumers to producers. Each peso transferred can be used to purchase $\phi_t$ goods in the centralized market (CM), thus saving the DM sellers from having to work $\dfrac{\phi_t}{A_t}$ hours during the CM. When $\theta = 1$, DM consumers have full bargaining power and make a take-it-or-leave-it offer to the producers equal to their cost of production $c(\frac{x}{A})$. Alternatively, when $\theta = 0$, the consumers pay for their consumption with a monetary payment valued at the total utility attained through DM consumption. In equilibrium, consumers will compensate producers for their labor by

[43] The equilibrium CM price level $\phi_t$ will be determined in the CM market.

paying them $\dfrac{\mu_t \phi_t}{A_t} = z(x_t, A_t)$. This condition equates the utility value of DM receipts (in terms of CM labor utility) to the seller's share of the surplus created from a DM match.

If the consumer is unconstrained by her money holdings, meaning that $\mu_t < m_t$ (and $\delta = 0$), it is easy to show using equations (8) and (9) that $u'(x) = (\frac{1}{A_t})c'(\frac{x}{A})$. The value of $x$ for which this result holds I denote as $x^{opt}(A_t)$. This optimal production level maximizes social surplus and represents the outcome that would result if the matching frictions did not exist in the decentralized market.[44] If $x^{opt}(A_t)$ results from the bargaining equilibrium, total social surplus is maximized. When consumers are not constrained by their money holdings, they are *able* to compensate producers to produce at this optimal level. However, as will be shown later, it will likely not be in the best interest of DM consumers to bring all of their money holdings into the decentralized market. I define $\mu^{opt}(A_t)$ as the "optimal payment" amount:

$$\mu^{opt}(A_t) = \left(\frac{A_t}{\phi_t}\right)\left[\theta c\left(\frac{x^{opt}(A_t)}{A_t}\right) + (1-\theta)u(x^{opt}(A_t))\right]. \tag{11}$$

If, however, consumers enter the DM with $m_t < \mu^{opt}(A_t)$ they will be unable to compensate producers for the optimal production level.[45] In these cases, producers will produce $x_t < x^{opt}(A_t)$ and receive $\mu_t = m_t$ in payment. The money constraint holds with equality and $\delta_t > 0$, yielding a solution that is defined by equation (3):

$$\mu_t = m_t = \left(\frac{A_t}{\phi_t}\right) z(x_t, A_t). \tag{12}$$

Equations (11) and (12) describe equilibrium conditions for any value of consumer money holdings, under the assumption that neither consumer nor producer optimality conditions bind with equality. Under these conditions, the DM market equilibrium choices for monetary transfers and production can be described as follows:

[44] This would be the equilibrium outcome if the DM behaved as a simple Walrasian market.

[45] Regardless of the slackness on the money transfer constraint, note that the DM equilibrium is independent of the seller's money holdings $\tilde{m}_t$.

$$\mu(m_t, S_t; \Omega) = \begin{cases} \mu^{opt}(A_t) & \text{if } m_t \geq \mu^{opt}(A_t) \\ m_t & \text{if } m_t < \mu^{opt}(A_t) \end{cases}$$

$$x(m_t, S_t; \Omega) \ \text{ solves} \begin{cases} u'(x) = (\frac{1}{A_t})c'(\frac{x}{A_t}) & \text{if } m_t \geq \mu^{opt}(A_t) \\ \frac{m_t \phi_t}{A_t} = z(x, A_t) & \text{if } m_t < \mu^{opt}(A_t) \end{cases}.$$

The seller's utility share of DM surplus $z(x_t, A_t)$ is an important component in describing the bargaining problem solution. Under the aforementioned assumptions about $u(x)$ and $c(h)$, it can be shown that $z_x(x, A) > 0$ *and* $x_m(m, .) = \dfrac{\phi}{\mu_m(m, .) A z_x(x, .)} > 0 \ \forall \ x \in (0, x_t^{opt})$ and $A > 0$. As consumers bring more money into the DM, the equilibrium negotiated level of DM consumption and the utility compensation received by the producers will both be greater. As $\mu_t \rightarrow \mu_t^{opt}$, it will also be the case that $x_t \rightarrow x_t^{opt}$ as well. Given these results, total social surplus is maximized in equilibria where $\mu_t = \mu^{opt}(A_t)$. Yet, this first best outcome will be unattainable as long as the DM consumer enters into the DM with less money than $\mu^{opt}(A_t)$.

Figure 14 shows how the solution to the bargaining problem, the $z$ function, changes with model parameters. In this figure, I graph the outcome $z$ function under the assumption that DM consumer preferences obey constant relative risk aversion (CRRA).[46] I present a baseline parameterization and then increase all relevant variables to show their effects on the bargaining problem equilibrium. The utility transfer $z$ is monotonically increasing in production $x$. As buyers' bargaining power grows, the equilibrium utility transfer decreases. When the CRRA utility variable $\sigma_{c,1}$ increases, the marginal utility of DM consumption rises and the DM consumers' willingness to pay for DM consumption both rise. In this iteration, the DM utility payment is larger for all production levels $x_t$. The last change increases the labor productivity $A$. Because the costs of production are lower, so too is the necessary amount of promised utility that must be given to the sellers to compensate for their effort.

[46] Let $u(x) = \frac{x^{1-\sigma_{c,1}}}{1-\sigma_{c,1}}$.

However, the optimal production level $x^{opt}(A_t)$ increases with the productivity level. When productivity is higher, ceteris paribus, the utility transfer per unit of production decreases, but the total amount of production and utility transferred to sellers may increase.

Having described the model equilibrium, I now return to check consumer and producer optimality conditions. First, consider the consumers' restriction that $u(x_t) \geq \frac{\phi_t \mu_t}{A_t}$. For this condition to hold, it must be the case that $u'(x_t) \geq z_x(x_t, A_t)$, meaning that the marginal increase in utility from buying an extra unit of DM consumption must be weakly greater than the marginal utility transferred to sellers in compensation for that unit. This is where the monetary hold-up problem manifests. The nature of the Nash bargaining problem is such that it may be in consumers' best interest to bring less money into the DM than they own. In other words, it may be worthwhile for consumers to leave some of their money at home if the relative benefit of spending this money in the DM is too low. Figure 15 presents $u'(x_t) - z_x(x_t, A_t)$ for the same parameterizations presented in Figure 16.

No solution to the bargaining problem exists for production levels of $x_t$ above which $u'(x_t) - z_x(x_t, A_t) = 0$. I define the value of $x$ that satisfies this equation as $\bar{x}_t$. If $x_t = \bar{x}_t$, then it also must be that $\xi_t \geq 0$. Returning to equation (8), one can calculate the equilibrium value of $\mu_t$ implied by this production level. When $\mu_t < m_t$, consumers limit the amount of money that they choose to bring with them into DM matches. This decision illustrates the nature of the hold-up problem, which is a direct result of Nash bargaining.[47]

As the amount of money the consumers bring into the DM increases, the relative marginal benefit of bringing a peso into the match declines. Eventually, it is no longer in the best interest of consumers to bring an additional unit of currency into the DM. As shown in Figure 15, consumers will not bring $\mu^{opt}(A_t)$ into a decentralized match even if $m_t > \mu^{opt}(A_t)$. As explained in LW, only in the case where $\theta = 1$ would the consumers ever choose $\mu_t = \mu^{opt}(A_t)$ when they have sufficient funds to do so. When $\theta = 1$, consumers reap the entire surplus

[47] See Aruoba et al. (2007) for a comprehensive discussion of the role of Nash bargaining in monetary search models. In general, the hold-up problem in LW models stems from 2 factors: non-monotonicity of Nash bargaining shares and inefficient money purchases attributable to the discount factor.

from the DM market match and therefore would seek the socially optimal outcome. When consumers have anything less than full bargaining power, they will always choose to enter the DM with money holdings less than $\mu^{opt}(A_t)$. As a result, the optimal DM outcome will be unattainable. Therefore, under Nash bargaining, $\bar{x}_t$ represents the feasible upper bound for DM production , where $\bar{\mu_t} \leq \mu^{opt}(A_t)$ is the corresponding maximum monetary transfer.

The complementary condition must be met for DM producers. The producer optimality condition implies that $(\frac{1}{A_t})c'(\dfrac{x_t}{A_t}) - z_x(x_t, A_t) \leq 0$ or that marginal benefit of producing an additional unit must be weakly greater than the marginal cost of producing said unit. In Figure 16, I present the DM terms of trade graph for producers. This diagram illustrates that for most parameter values there will exist no lower bound on production. It will always be in the best interest of producers to generate goods at the equilibrium given by the Nash bargaining problem in equation (2). A complete description of the Nash Bargaining equilibrium outcome is:

$$\mu(m_t, S_t; \Omega) = \begin{cases} \bar{\mu_t} & \text{if} m_t \geq \bar{\mu_t} \\ m_t & \text{if } m_t < \bar{\mu_t} \end{cases}$$

$$x(m_t, S_t; \Omega) \ \text{ solves} \begin{cases} \frac{\bar{\mu_t}\phi_t}{A_t} = z(x, A_t) & \text{if } m_t \geq \bar{\mu_t} \\ \frac{m_t\phi_t}{A_t} = z(x, A_t) & \text{if } m_t < \bar{\mu_t} \end{cases}.$$

**Centralized Market Solution**

Given the results from the bargaining problem, I return to the value function

$$V(m_t, S_t; \Omega) = \nu(m_t, S_t; \Omega) + \frac{\phi_t m_t}{A_t} + \frac{T_t}{A_t} + \max_{m_{t+1}} \left\{ -\frac{\phi_t m_{t+1}}{A_t} + \beta E \left( V(m_{t+1}, S_{t+1}; \Omega) \right) \right\}$$

where

$$
\begin{aligned}
\nu(m_t, S_t; \Omega) \ \ &\equiv \ \ \sigma \left[ u(x(m_t, S_t; \Omega)) - \frac{\phi_t \mu(m_t, S_t; \Omega)}{A_t} \right] \\
&\quad + \sigma \int \left[ -c\left( \frac{x(m_t, S_t; \Omega)}{A_t} \right) + \frac{\phi_t \mu(\tilde{m}_t, S_t; \Omega)}{A_t} \right] dF_t(\tilde{m}) \\
&\quad + U(X^{opt}) - \frac{X^{opt}}{A_t}.
\end{aligned}
$$

Maximizing with respect to $m_{t+1}$ together with the envelope condition for $m_t$ yields the necessary condition for a maximum:

$$
-\frac{\phi_t}{A_t} + \beta E_t \left( \sigma_m \left[ u_x(x_{t+1}) x_m(m_{t+1}, .) - \frac{\phi_{t+1}}{A_{t+1}} \mu_m(m_{t+1}, .) \right] + \frac{\phi_{t+1}}{A_{t+1}} \right) \leq 0. \qquad (13)
$$

The above condition in conjunction with requisite second-derivative conditions is sufficient to prove the existence of a maximum. Ensuring a finite maximum requires that $\frac{\phi_t}{A_t} \geq \beta E_t \left( \frac{\phi_{t+1}}{A_{t+1}} \right)$. Derived from the first-order condition for money holdings, this asserts that the expected discounted future value of money cannot exceed the utility value of money in the current period. If this condition were not met, agents would demand an infinite amount of money in the current period. In equilibrium, the value of money today will adjust so that this condition is always satisfied. As long as money demand is finite, the objective function will be concave with respect to $m_{t+1}$ over the feasible domain, yielding a unique maximum. Additionally, given that the CM solution is independent of incoming money holdings and the existence of a utility-maximizing value for $m_{t+1}$, quasi-linear preferences yield a *unique* choice of money holdings $m_{t+1}$ for all agents in equilibrium.[48] This monetary equilibrium must then result in $m_{t+1} = M_{t+1}$ with a degenerate $F_{t+1}$. Focusing on degenerate equilibria and then plugging in for $x_m$, it follows that when a monetary equilibrium exists, the centralized market solution can be written as:

[48] This requires ensuring that $H \geq 0$ for DM sellers.

$$\frac{\phi_t}{A_t} = \beta E_t \left[ \left( \frac{\phi_{t+1}}{A_{t+1}} \right) \left( \sigma_m \mu_m(m_{t+1}, .) \left[ \left( \frac{u'(x_{t+1})}{z_x(x_{t+1}, A_{t+1})} \right) - 1 \right] + 1 \right) \right]. \tag{14}$$

## Defining the Model Equilibrium

A recursive competitive equilibrium is a set of stationary household decision rules and transition equations that satisfy utility maximization, budget constraints and market-clearing conditions for all agents in the economy, given a government policy function. Let $S_t \equiv (A_t, b_t, b_t^*, M_t)$ be the aggregate state of the economy.

I define the function $\Omega(S_t) = \{g_t,\ d_t,\ d_t^*,\ b_{t+1},\ b_{t+1}^*\}$ as the government's policy function, mapping the state of the world into the government's choice set. Given the government's policy function, I can define $F_{t+1} = \Gamma(S_t; \Omega)$ as the law of motion for the distribution of money holdings, and $\phi_t = \Lambda_\phi(S_t; \Omega)$ as the law of motion for CM prices, $q_t^i = \Lambda_q^i(S_t; \Omega)$ as the law of motion for bond prices of types $i \in \{domestic,\ foreign\}$. Additionally, let $b_{t+1}^i = \Upsilon_{b^i}(S_t; \Lambda_{q^i}, \Omega)$ be the foreign investors' bond demand functions. Then household decision functions (for DM buyers and sellers, indexed by $j$) are money holdings $m_{j,t+1} = m_j(m_j, S_t; \Omega)$, CM production $X_{j,t} = X_j(m_{j,t}, S_t; \Omega)$ and DM exchange functions $\mu_{j,t+1} = \mu_j(m_{j,t}, S_t; \Omega)$ and DM production $x_{j,t} = x_j(m_{j,t}, S_t; \Omega)$.

**Definition 1** *Recursive Competitive Equilibrium.* A recursive competitive equilibrium is a law of motion for money holdings $\Gamma$, a CM price function $\Lambda_\phi$, pricing functions $\{\Lambda_\phi, \Lambda_{q^i}, \Upsilon_{b^i}\}$, value functions $\{V, W\}$, DM exchange variables $\{x_t, d_{j,t}\}$ and CM exchange variables $\{X_j, m_j\}$ for a given policy function $\Omega$, such that:

1. Values and quantities in the centralized market solve (3), taking prices and aggregate functions as given.

2. Decentralized market quantities solve the symmetric Nash bargaining problem in (1), taking price and aggregate functions as given.

3. The domestic money market clears, meaning $M_{t+1} = \sigma_m \int m_{b,t+1}(m_{b,t})dF_t(m_{b,t}) + \sigma_m \int m_{s,t+1}(m_{s,t})dF_t(m_{s,t})\ \forall t$.

4. All goods markets clear $\forall t$.

5. Bond markets clear $\forall t$.

6. The government and agent budget constraints are satisfied.

## Markov-Perfect Equilibrium

Although many equilibria could exist for this particular problem, I choose to analyze problems with a stationary equilibrium government policy function. Here the government chooses a function $\Psi(S_t;\Omega) = \omega_t$ that maps its future policy rule and current state into a current policy rule to maximize its current period objective function. I denote the value functions in equilibrium as $\{\overline{V}(m_t, S_t, \omega_t;\Omega), \overline{W}(m_t, S_t, \omega_t;\Omega)\}$ and the government's value function as $\overline{V}^g(S_t, \omega_t;\Omega)$ to differentiate from those described in the definition for the recursive competitive equilibrium.[49] To find a stationary policy function, I will restrict my search to policy functions that do not depend on past histories. Additionally, the equilibrium policy function must also satisfy $\omega_t = \Psi(S_t, \Omega)$. Therefore, I define this equilibrium as follows:

**Definition 2** *Markov-Perfect Equilibrium.* A Markov-perfect equilibrium is:

1. A law of motion for money holdings $\{\Gamma\}$, a CM price function $\{\Lambda_\phi\}$, pricing functions $\{\Lambda_\phi, \Lambda_{q^i}, \Upsilon_{b^i}\}$, value functions $\{V, W, V^g\}$, DM exchange variables $\{x_t, d_{j,t}\}$, and CM exchange variables $\{X_j, m_j\}$ that satisfy the definition of a recursive competitive equilibrium.

2. A law of motion for money holdings $\{\overline{\Gamma}\}$, a CM price function $\{\overline{\Lambda}_\phi\}$, pricing functions $\{\overline{\Lambda}_\phi, \overline{\Lambda}_{q^i}, \overline{\Upsilon}_{b^i}\}$, value functions $\{\overline{V}, \overline{W}, \overline{V}^g\}$, DM exchange variables $\{\overline{x}_t, \overline{d}_{j,t}\}$, and CM exchange variables $\{\overline{X}_j, \overline{m}_j\}$, which all satisfy the definition of a recursive competitive

[49] All decision rules and laws of motion that satisfy this equilibrium will be written with overbars.

equilibrium. Current period functions are denoted with bars, while all subsequent periods are defined as in definition 1.

3. A current policy function $\Psi(S_t, \Omega)$ that maximizes $\overline{V}^g(S_t, \omega_t; \Omega)$ in the recursive competitive equilibrium, which coincides with $\Omega$ the future period policy function such that:

$$\Omega(S_t) = \Psi(S_t, \Omega) = \omega_t = arg \max_{\hat{\omega}_t} \overline{V}^g(S_t, \hat{\omega}_t; \Omega).$$

## Proving the Existence of a Unique Solution for the Choice of Money Holdings

The derivative of the objective function with respect to choice of money in the current period is described in equation 6. However, this condition is not sufficient to guarantee the existence of a maximum for $m_{t+1} > 0$. Second-order conditions must also be satisfied to ensure that a maximum does indeed exist. The necessary second-order condition with respect to money choice is:

$$\sigma_m \left[ u''(x) x_m(m_{t+1})^2 - u'(x) x_{mm}(m_{t+1}) \right] \leq 0.$$

If $m_{t+1} \geq \mu^{opt}(A_{t+1})$, then $x_m(m_{t+1})^2 = 0$ and $x_{mm}(m_{t+1}) = 0$, satisfying the above condition. However, if $m_{t+1} < \mu^{opt}(A_{t+1})$, it is the case that $x_m(m_{t+1})^2 = 1$ and the above equation can be rewritten:

$$[u''(x) + u'(x) x_{mm}(m_{t+1})] \leq 0.$$

Then, substituting for $u'(x) x_{mm}(m_{t+1})$:

$$\left[ u''(x) + \frac{\phi_{t+1} z_{xx}}{A z_x^2} \right] \leq 0.$$

Given the assumption that $u'' > 0$, this requires proving that $z_{xx} < 0$. Although not analytically feasible, numerical simulations show that the $z$ function is indeed concave with respect to DM production, and thus the objective function is weakly concave with respect to all feasible money choices. Then, if equation (3) holds with equality, it can be shown that an interior maximum exists for $m_{t+1}$.

## Ensuring Degenerate Monetary Distribution

To ensure that a degenerate distribution exists, it must be the case that in equilibrium, sellers always use the entirety of their monetary earnings from the DM to purchase CM goods from sellers. If sellers do not use all of their revenues in the CM then, they will necessarily leave the DM with more money than their buyer counterparts.

This restriction limits the sovereign's net borrowing every period. If the sovereign borrows on net a sufficiently large sum, say $X^{opt}(A_t)$, and transfers that amount to all agents, the sellers in the DM will have no incentive to spend any of their money in the CM. Thus, focusing on degenerate distributions requires:

$$\left(X^{opt}(A_t) - T_t\right) \geq \phi_t \mu_t.$$

Expanding $T_t$ and assuming no default:

$$\left(X^{opt}(A_t) - \phi_t\left(q_t b_{t+1} - b_t\right) - \phi_t^*\left(q_t^* b_{t+1}^* - b_t^*\right)\right) \geq \phi_t \mu_t.$$

Note that bond revenues are functions of current period variables and known in the current period. I can then substitute:

$$brev_t \equiv \phi_t q_t b_{t+1} + \phi_t^* q_t^* b_{t+1}^*$$

Then we have:

$$\left(X^{opt}(A_t) - brev_t + \phi_t b_t + \phi_t^* b_t^*\right) \geq \phi_t \mu_t$$

$$\left(X^{opt}(A_t) - brev_t + \phi_t^* b_t^*\right) \geq \phi_t \left(\mu_t - b_t\right)$$

And using the monetary normalization:

$$\left(X^{opt}(A_t) - brev_t + \phi_t^* b_t^*\right) \geq \frac{\hat{\phi}_t}{g_t} \left(\hat{\mu}_t - \hat{b}_t\right)$$

I define:

$$\eta_t \equiv X^{opt}(A_t) - brev_t + \phi_t^* b_t^*$$

The restrictions of the money growth rate $g_t$ can be described as follows:

$$\text{If } \eta_t \leq 0 \begin{cases} g_t \leq \dfrac{\phi_t\left(\hat{\mu}_t - \hat{b}_t\right)}{\eta_t} & \text{if } \hat{\phi}_t\left(\hat{\mu}_t - \hat{b}_t\right) \leq 0 \\ g_t \leq 0 & \text{if } \hat{\phi}_t\left(\hat{\mu}_t - \hat{b}_t\right) > 0 \end{cases}$$

$$\text{If } \eta_t > 0 \begin{cases} g_t > 0 & \text{if } \hat{\phi}_t\left(\hat{\mu}_t - \hat{b}_t\right) \leq 0 \\ g_t > \dfrac{\phi_t\left(\hat{\mu}_t - \hat{b}_t\right)}{\eta_t} & \text{if } \hat{\phi}_t\left(\hat{\mu}_t - \hat{b}_t\right) > 0. \end{cases}$$

Note that when both $\eta \leq 0$ and $\phi_t\left(\hat{\mu}_t - \hat{b}_t\right) \leq 0$, there can exist no degenerate monetary equilibrium. In these cases, sovereign borrowing exceeds the real value of the DM monetary transfer for any growth rate of money $g > 0$. In all other cases, a degenerate monetary equilibrium may (or will always) exist for some $g > 0$.

Everything above follows in a defaulting period, simply with the sovereign paying a haircut and not borrowing in the current period. Typically, defaulting periods will occur when debt is high and borrowing capabilities are low, therefore making it unlikely to be a cause of concern.

## Proof of Proposition 1

Consider the value function of this planner's problem in this model:

$$V(S_t) = \max_{x_t, X_t} \left\{ \sigma_m \left( u(x_t) - c(\frac{x_t}{A_t}) \right) + BU(X_t) - \frac{X_t}{A_t} + \beta E_t \left[ V(S_{t+1}) \right] \right\}$$

The production values that satisfy this maximization are $x^{opt}(A_t)$ and $X^{opt}(A_t)$. Additionally, as shown previously, the value function is increasing in $x_t$ for all $x_t \leq x^{opt}(A_t)$. When considering the competitive problem, the planner's solution to the CM, $X^{opt}(A_t)$, is always achieved. To maximize CM surplus, the government will optimally choose a monetary policy to support the highest level of $x_t$, not to exceed $x^{opt}(A_t)$. The highest feasible value for $x_t$ when Nash Bargaining takes place in the CM will be $\bar{x}(A_t)$. A government policy that sets $g_t = \bar{g}(S_t)$, where $\bar{g}(S_t)$ is defined as the largest rate of money growth that supports $x^{opt}(A_t)$ in equilibrium, will therefore satisfy the government's problem and maximize social surplus.

## Proof of Proposition 2

Quasi-linearity of CM labor costs implies that $X_t = X_t^{opt}(A_t)$ $\forall t$ and that there are no wealth effects for agents in the CM. In addition, sovereign borrowing in the CM serves only to reduce CM labor burdens. As long as the degeneracy of money holdings is maintained, money growth choices will be unaltered by incoming debt levels. Therefore, the value function of a sovereign in default is defined as $V^{Def}$ and the corresponding non-default value function is defined as $V^{ND}$, then the difference between these two functions can be defined as:

$$V^{ND}(b_t^*, .) - V^{Def}(b_t^*, .) = \beta E_t[V^{ND}(b_{t+1}^*, .) - V^{ND}(0, .)] + (q_t^* \phi_t^* b_{t+1}^* - \tau) + \phi_t^* b_t^* (\gamma - 1).$$

When $V^{ND}(b_t^*, .) - V^{Def}(b_t^*, .) < 0$, default will be preferred to repayment. Note that $q_t^*$ is a function only of choice variables and not a function of $b_t^*$. Thus, if there exists $b_{def}^*(A)$ such that $V^{ND}(b^*_{def}(A), .) - V^{Def}(b^*_{def}(A), .) = 0$, and $\gamma < 1$, then it must be the case that for any $b_t > b^*_{def}(A)$, default must be strictly preferred to debt repayment.

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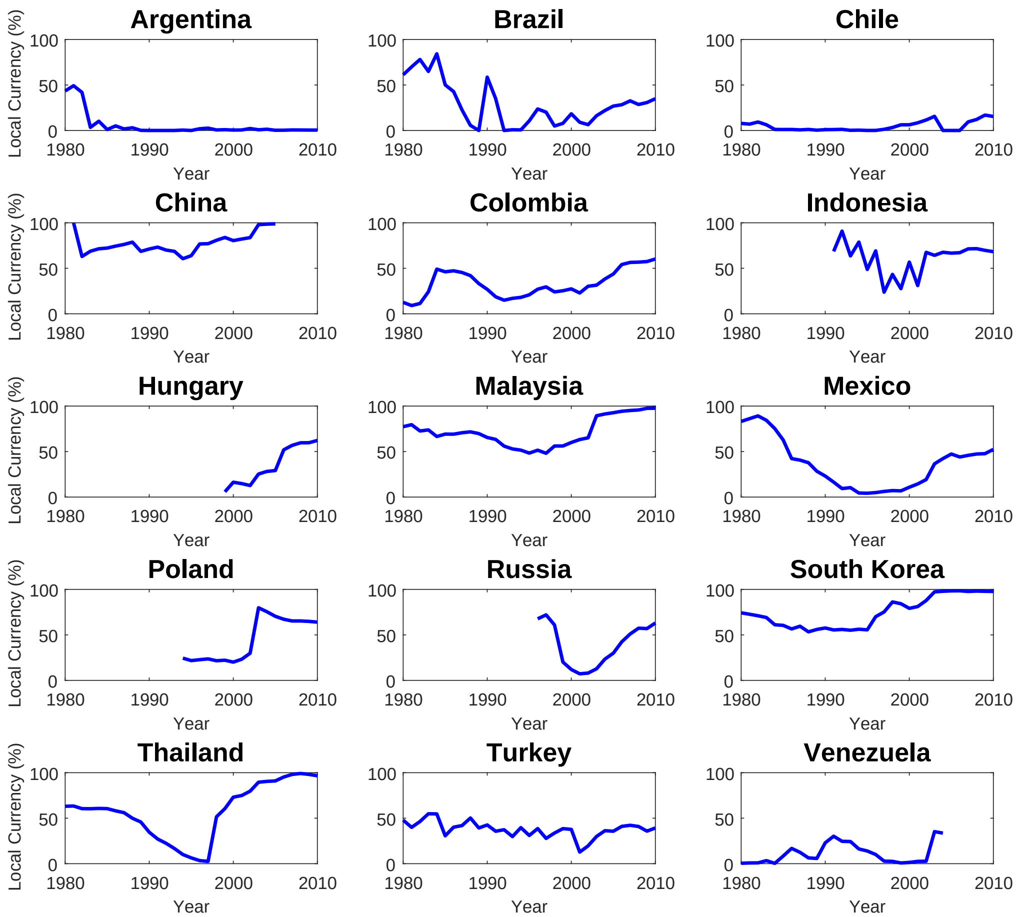
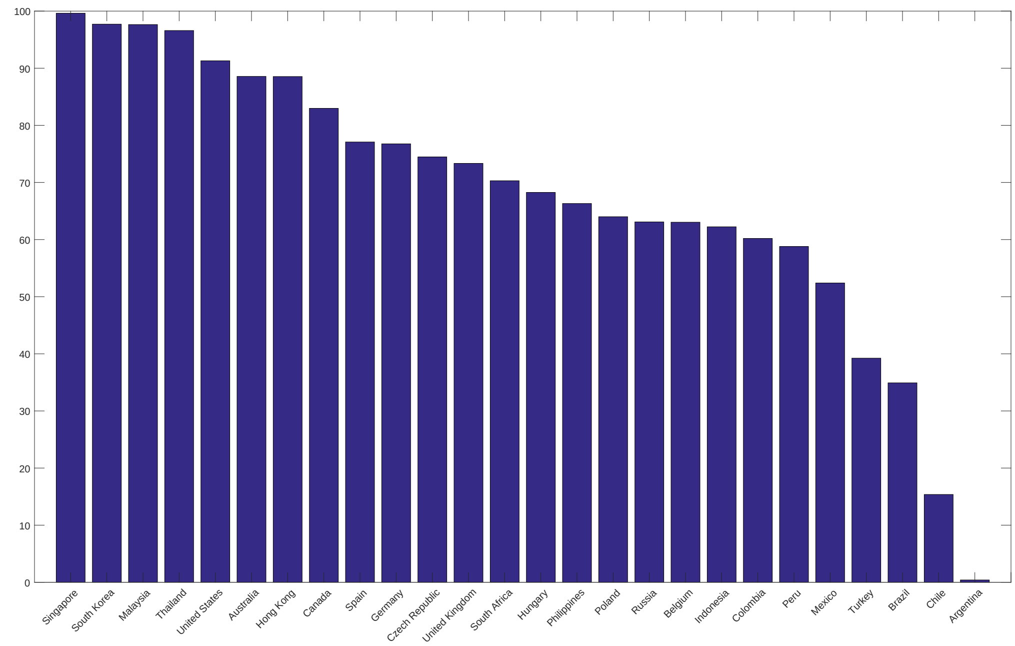
# Figures and Tables

Figure 1: Percentage of Total Sovereign Debt Denominated in Local Currency



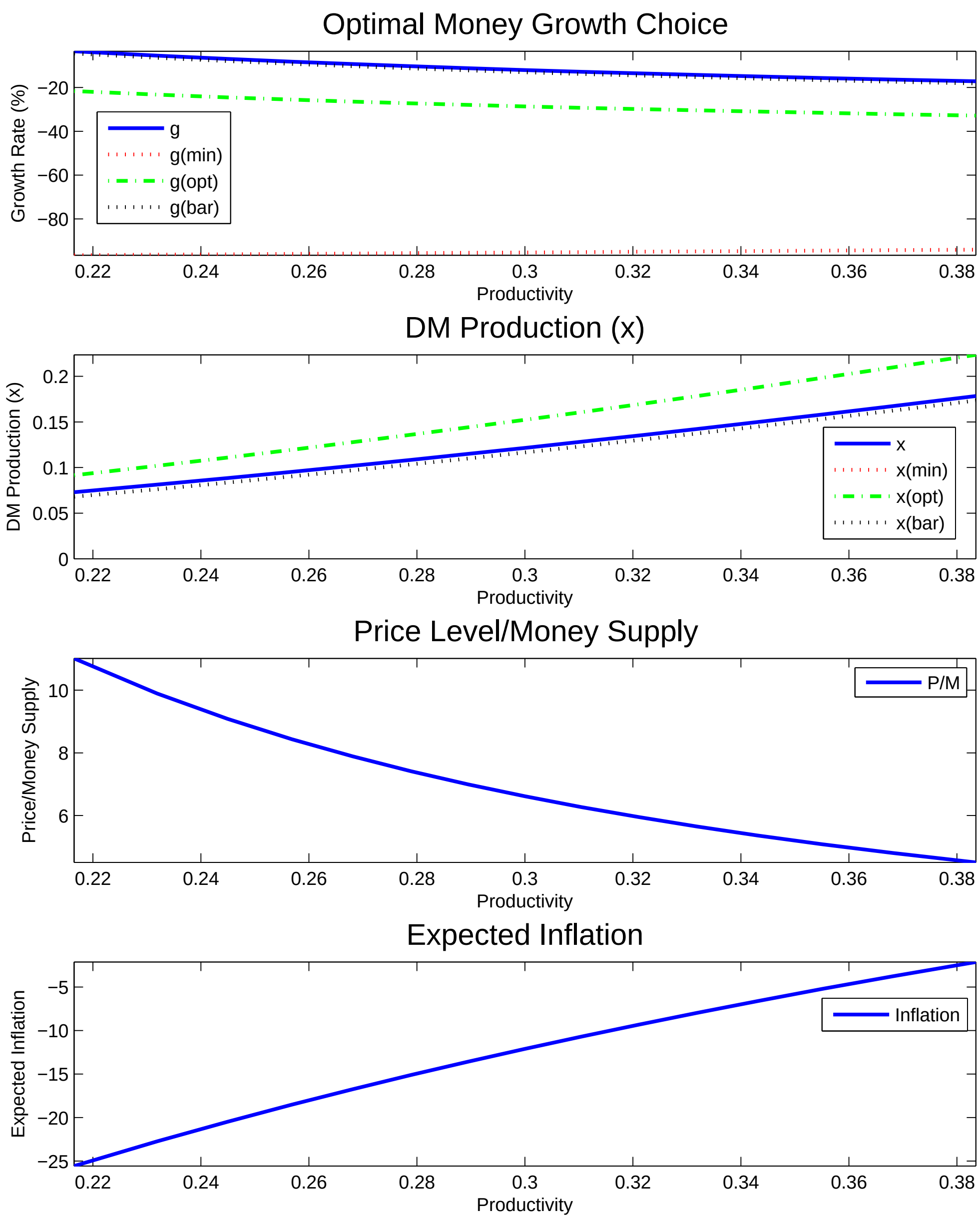

Note: This figure presents the percentage of total sovereign debt denominated in local currency for a selection of emerging market countries. Local currency sovereign debt is considered debt denominated in the local currency with a fixed interest rate. This does not include debt that is denominated in local currency with a variable interest rate or that is indexed either to another currency or to inflation.
Data sources: BIS Public Debt Data, Jeanne and Guscina (IMF) dataset, and the CLYPS dataset.

Figure 2: Percentage of Total Sovereign Debt Denominated in Local Currency, 2010



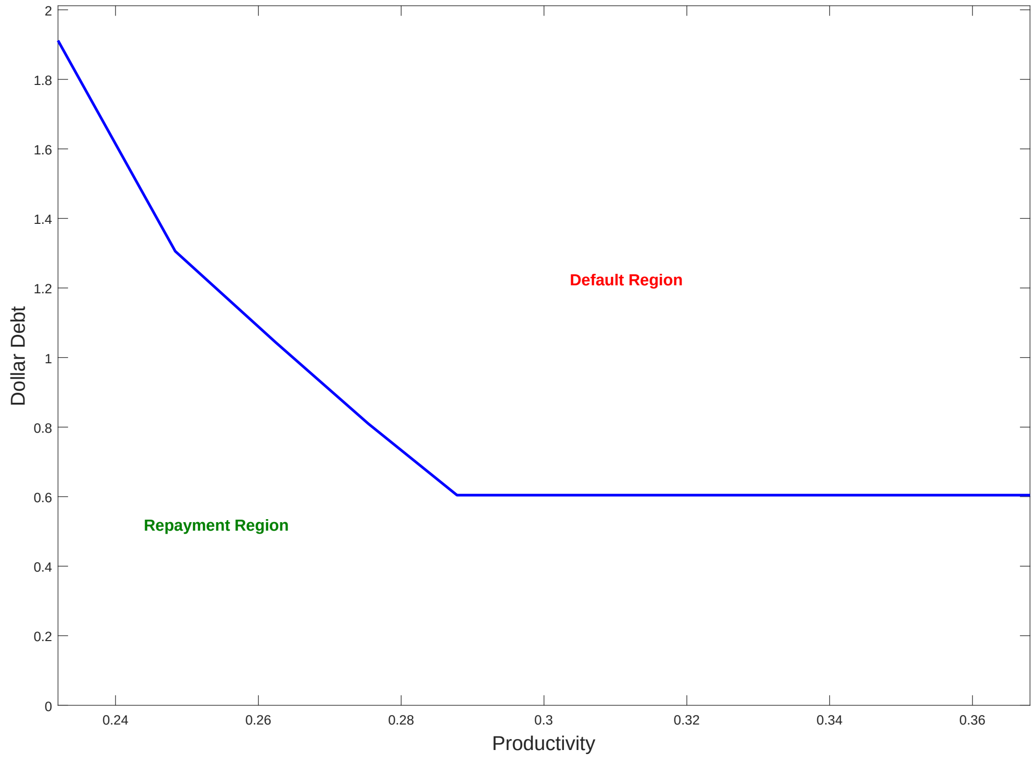

Note: This figure presents the percentage of total sovereign debt denominated in local currency for a wide range of countries for the year 2010. Local currency sovereign debt is defined as debt denominated in the local currency with a fixed interest rate. This does not include debt that is denominated in local currency with a variable interest rate or that is indexed either to another currency or to inflation.
Data sources: BIS Public Debt Data, Jeanne and Guscina (IMF) dataset, and the CLYPS dataset.

Figure 3: Money Growth Policy Function



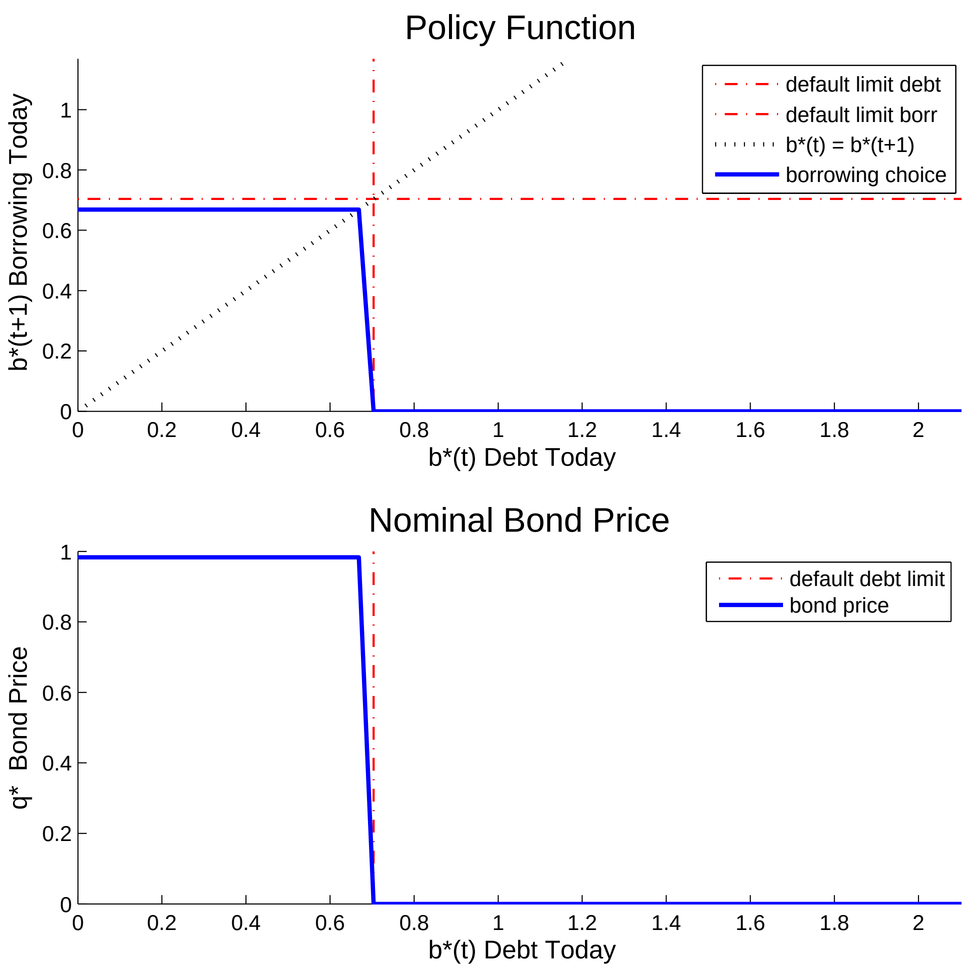

Note: This figure shows the decision rule of the government without access to debt. In the graphs, the government can vary only the growth rate of money $g_t$. The upper panel shows the optimal money growth rate, along with relevant money growth thresholds. The lower panels display consumption, the price to money supply ratio, and expected inflation.

Figure 4: Dollar Debt Default Threshold



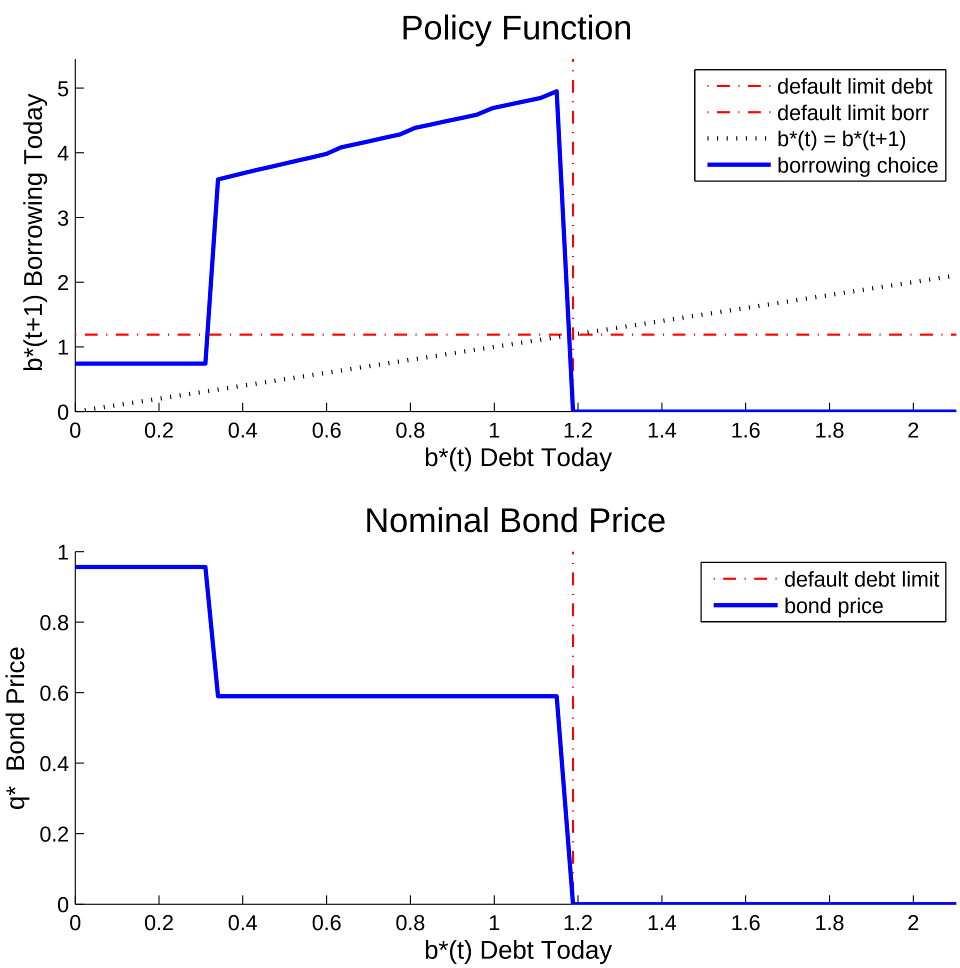

Note: This figure shows how the default threshold varies by productivity level. The sovereign chooses to default on any incoming debt level above this default threshold.

Figure 5: Dollar Debt Policy Function: High Productivity



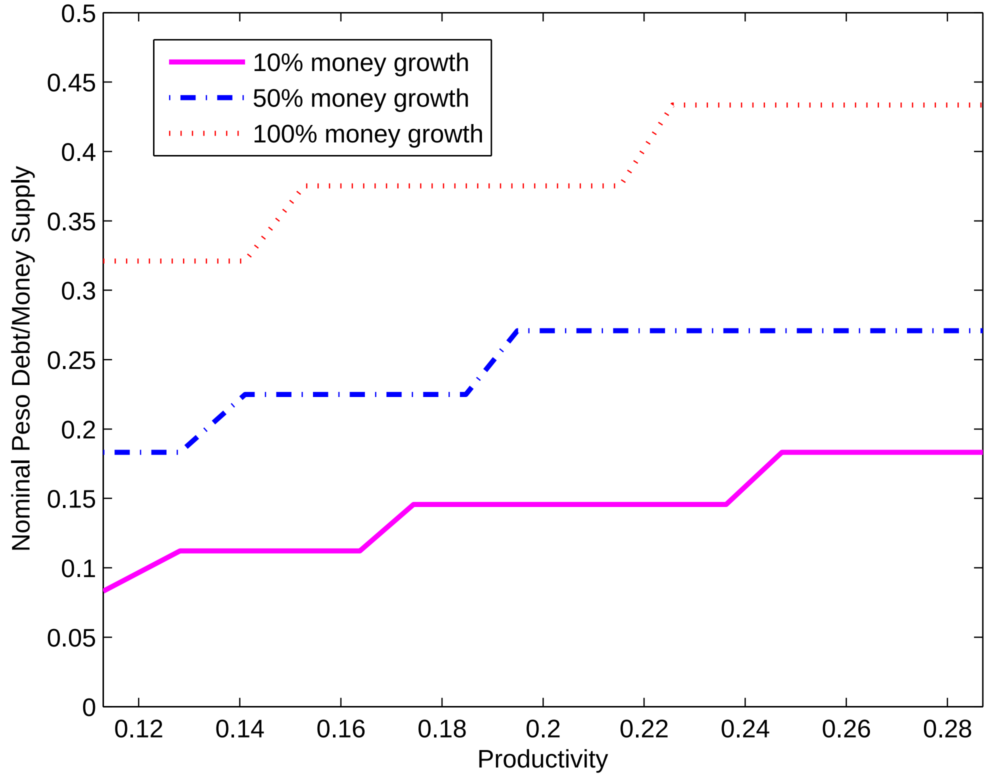

Note: This figure presents the policy function for the sovereign, given the incoming debt level $b_t^*$ when productivity is high. The top panel shows borrowing $b_{t+1}^*$ as a function of the amount of debt the government enters the period with $b_t^*$. The bottom panel shows the implied bond price from borrowing $b_{t+1}^*$, given the incoming level of foreign currency debt.

Figure 6: Dollar Debt Policy Function: Low Productivity



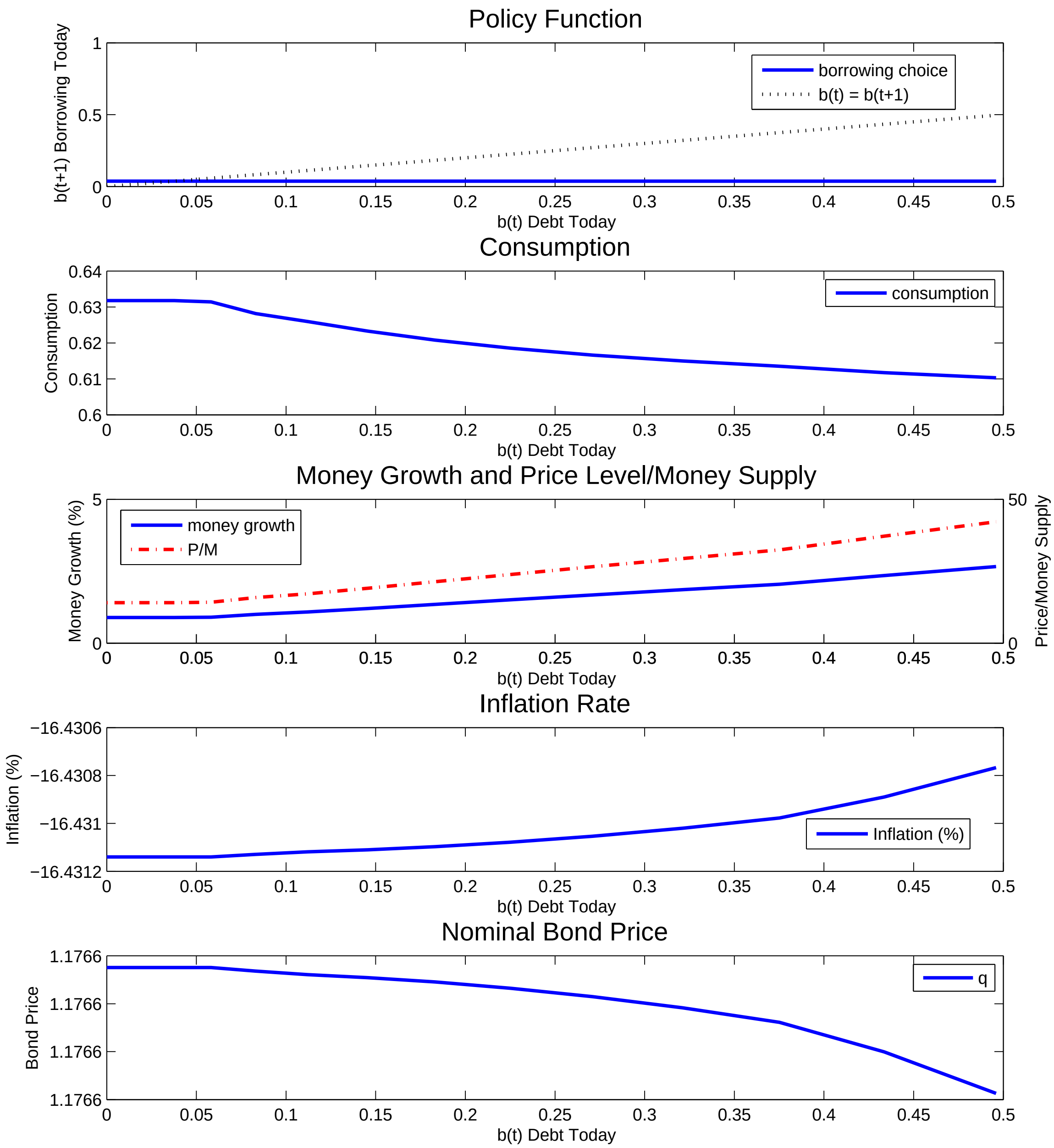

Note: This figure presents the policy function for the sovereign, given the incoming debt level $b_t^*$ when productivity is low. The top panel shows borrowing $b_{t+1}^*$ as a function of the amount of debt the government enters the period with, $b_t^*$. The bottom panel shows the implied bond price from borrowing $b_{t+1}^*$, given the incoming level of foreign currency debt.

Figure 7: Peso Debt Money Growth Choices



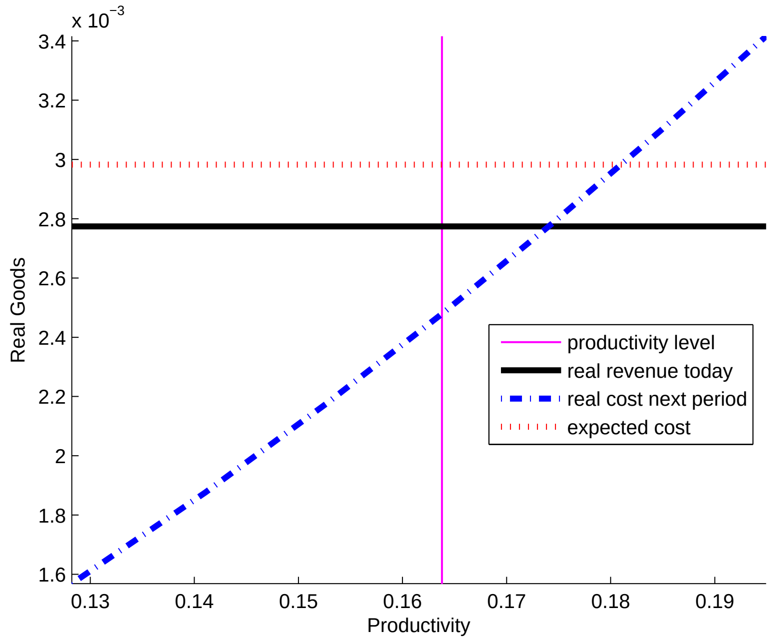

Note: This figure shows how money growth choices vary by productivity level and by the amount of local currency debt. Each line in this diagram presents a money growth rate. As debt increases, optimal money growth rates rise. The pink line shows where optimal growth rates exceed 10 percent. The blue dashed line shows where optimal money growth exceeds 50 percent. Lastly, the red dotted line shows where the optimal money growth rate passes 100 percent.

Figure 8: Peso Bond Policy Function



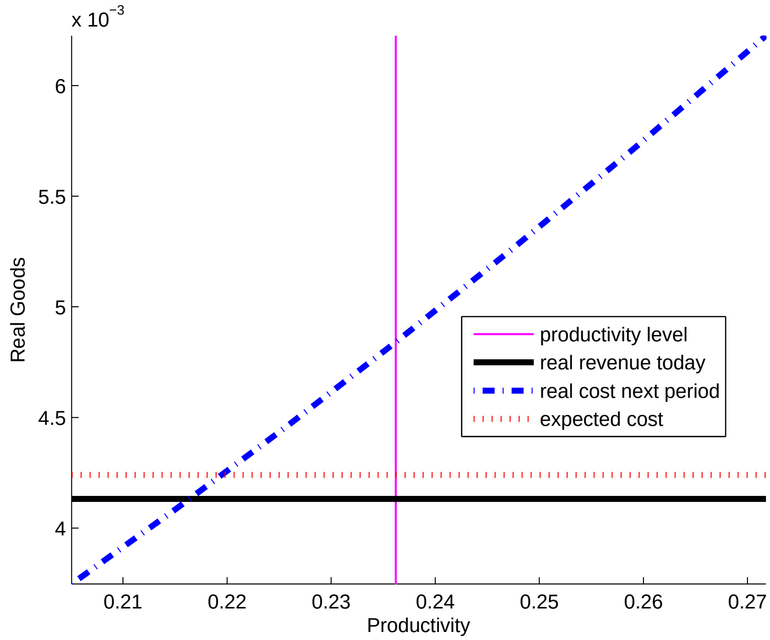

Note: This figure presents the policy function for the sovereign, given the incoming debt level $b_t$. Panels shows borrowing $b_{t+1}$, consumption, money growth, the price level to money supply ratio, the inflation rate, and the nominal bond price (given the borrowing choice) as a function of the amount of debt the government enters the period with $b_t$.

Figure 9: Peso Bond Payoff Function: Low Productivity



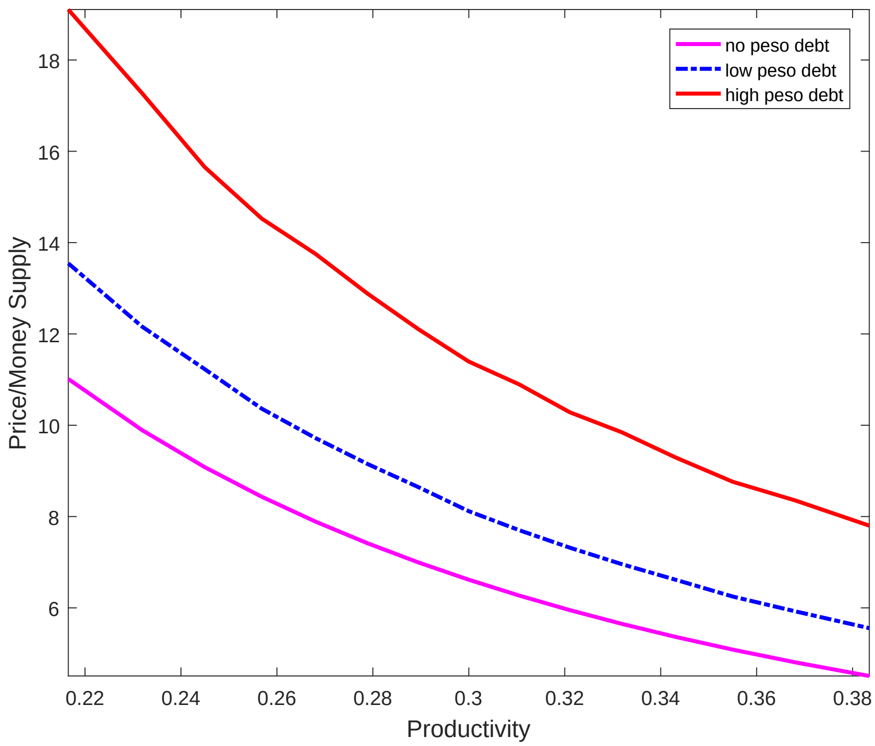

Note: This figure shows how local currency bond payoffs differ, depending on the future productivity realization (when the current period productivity level is low). The pink vertical line in the figure shows the current period productivity level. At this productivity level, the sovereign borrows real CM goods equal to the black horizontal line. The red dotted line shows the expected real repayment in the next period. Note that the difference between the two lines represents the real interest rate. The blue dotted line shows the realized cost of borrowing in the future period if future productivity corresponds to the horizontal axis productivity level.

Figure 10: Peso Bond Payoff Function: High Productivity



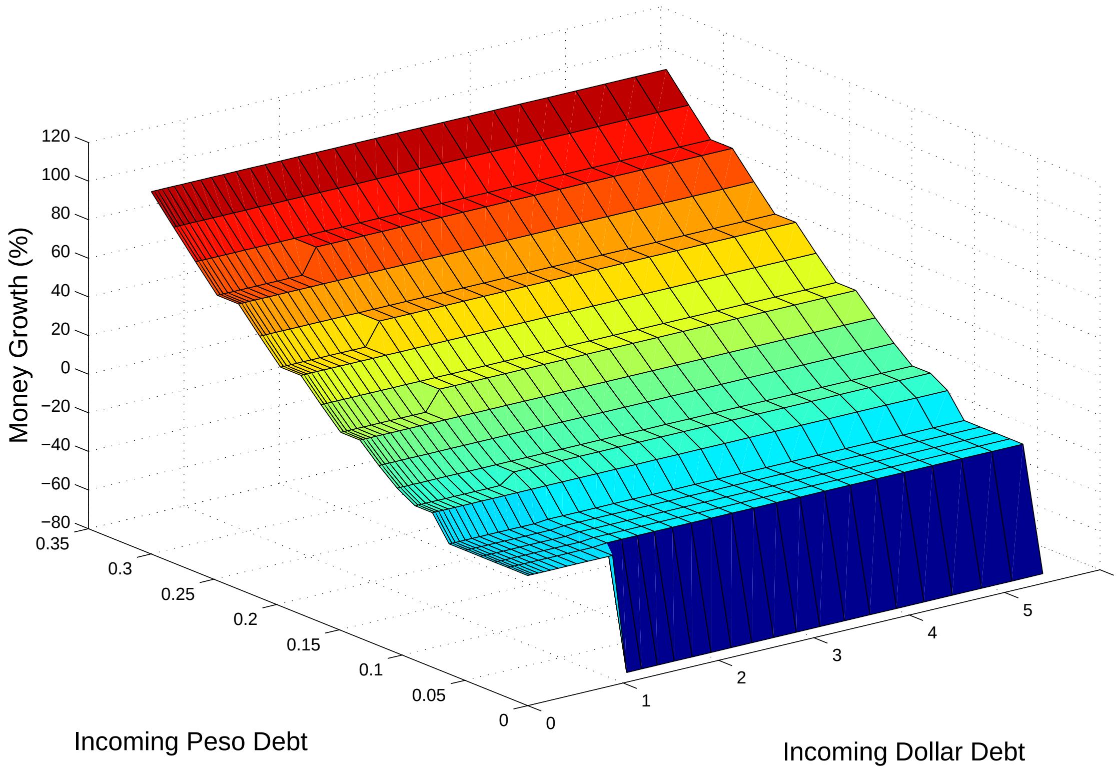

Note: This figure shows how local currency bond payoffs differ, depending on the future productivity realization (when the current period productivity level is high). The pink vertical line in the figure shows the current period productivity level. At this productivity level, the sovereign borrows real CM goods equal to the black horizontal line. The red dotted line shows the expected real repayment in the next period. Note that the difference between the two lines represents the real interest rate. The blue dotted line shows the realized cost of borrowing in the future period if future period productivity corresponds to the horizontal axis productivity level.

Figure 11: Price Function Manipulation: Peso Debt Model



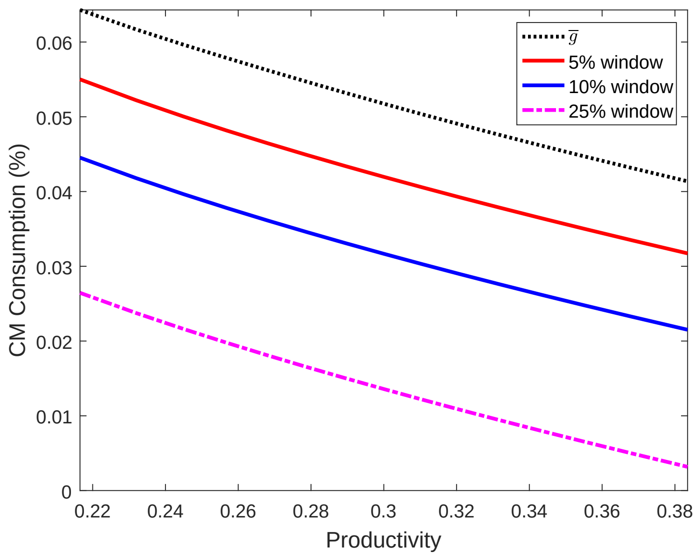

Note: This figure shows the price function by productivity for three levels of incoming local currency debt: no debt, low debt, and high debt.

Figure 12: Money Choice by Incoming Debt Level



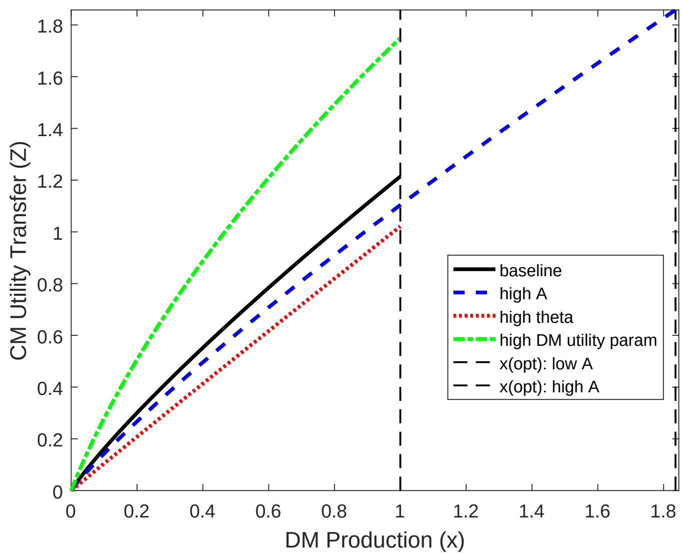

Note: This figure shows optimal money growth choices in a period as a function of incoming debt levels.

Figure 13: Welfare Comparison: Restricted and Unrestricted Monetary Policy



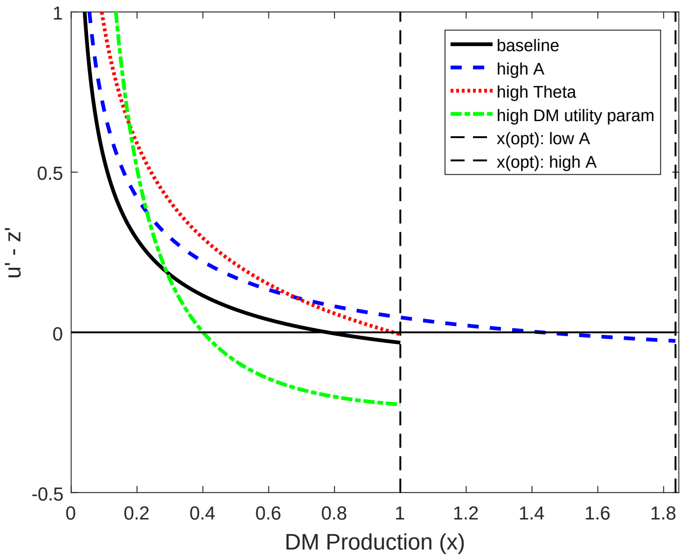

Note: This figure describes welfare differences in variations of the model with varying levels of monetary policy commitment. At each productivity level, I present the difference in welfare between the benchmark model (where the government has complete discretion over monetary policy choices) to models where the government is restricted to choosing the rate of money growth in some window around the efficient money growth choice $g_t = \bar{g}$. Welfare margins are calculated using averages over simulations, and welfare differences are calculated in terms of the annual percentage change in CM consumption utility.

Figure 14: Bargaining Problem Solution



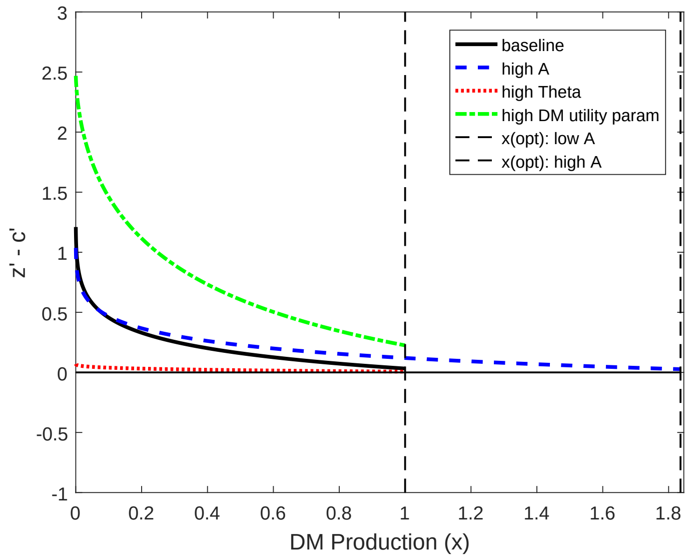

Note: This figure presents the seller's share of DM generated utility as a function of the production/consumption level $x$.

Figure 15: Consumer Terms of Trade




Note: This figure presents a measure of decentralized market consumer terms of trade. Graphed lines show the difference between the marginal cost of consumption and the marginal utility of the DM transfer to the producer for all feasible production levels $x$. Where these lines fall below zero, the consumer will not want to engage in trade.

Figure 16: Producer Terms of Trade




Note: This figure presents a measure of decentralized market producer terms of trade. Graphed lines show the difference between the marginal utility of the DM transfer to the producer and the marginal cost of production for all feasible production levels $x$.

## Table 1: Effect of Parameters on Model Moments

| | $\bar{A}$ | $\beta$ | $B$ | $\sigma_{c_1}$ | $\sigma_m$ | $\tau$ | $\theta$ |
|---|---|---|---|---|---|---|---|
| Inflation | -0.72 | 0.11 | -0.06 | -0.01 | 0.38 | 0.23 | 0.05 |
| Velocity | -0.07 | -0.06 | -0.01 | -0.13 | -0.09 | -0.09 | 0.07 |
| Retail Markup | 0.00 | -0.08 | 0.01 | 0.79 | -0.03 | -0.03 | -0.48 |
| Money Growth | -0.73 | 0.07 | -0.06 | -0.03 | 0.38 | 0.19 | 0.06 |
| Price/Money | -0.06 | -0.08 | -0.05 | -0.12 | -0.09 | -0.09 | 0.07 |
| % CM Cons. | -0.46 | -0.01 | 0.35 | -0.48 | -0.46 | -0.01 | 0.10 |
| $x$ | 0.72 | 0.06 | -0.01 | 0.66 | 0.01 | 0.02 | 0.03 |
| $d^*$ | -0.10 | -0.16 | 0.21 | 0.00 | 0.00 | -0.48 | 0.00 |
| $d$ | 0.04 | -0.04 | 0.05 | 0.04 | 0.04 | -0.04 | -0.04 |
| % Peso Debt | -0.06 | -0.14 | 0.18 | 0.07 | 0.03 | -0.53 | -0.03 |
| Debt/GDP | 0.04 | -0.09 | -0.27 | -0.04 | -0.04 | 0.84 | 0.01 |

Note: This table presents comparative statics for the model's fitted parameters. The model was simulated for an array of possible values for the fitted parameters. For each simulation, model moments were calculated. The elements in the above table represent the correlations between the fitted parameters and model moments.

## Table 2: Calibration Results

| | Model Moment | Data Moment | Parameter | Value |
|---|---|---|---|---|
| Default Freq. (%) | 2.94 | 4.91 | $\bar{A}$ | 0.30 |
| Debt/GDP | 0.33 | 0.39 | $\beta$ | 0.88 |
| Inflation (%) | -10.71 | 26.78 | $B$ | 74.20 |
| Monetary Velocity | 20.52 | 11.82 | $\sigma_m$ | 0.36 |
| Interest Rate on $ Denom. Debt (%) | 3.13 | 6.19 | $\tau$ | 1.02 |
| Retail Markup (%) | 24.33 | 30.00 | $\theta$ | 0.87 |
| *Debt in Dom. Curr. (%)* | 2.94 | 34.55 | $\sigma_{c_1}$ | 0.64 |

Note: This table presents results of the calibration procedure. The final row, *Debt in Dom. Curr. (%)*, was not a calibrated model moment. This statistic is displayed to present the model's implication for the key statistic in this paper, the currency composition of sovereign debt.

## Table 3: Model Correlations

| | $A$ | GDP | Money Growth | Inflation | % Peso Debt | Debt/GDP | Peso IR Spread | $ IR Spread | Default |
|---|---|---|---|---|---|---|---|---|---|
| $A$ | 1.00 | 1.00 | -0.99 | -0.50 | -0.06 | -0.14 | 1.00 | -0.05 | -0.13 |
| GDP | | 1.00 | -0.99 | -0.50 | -0.06 | -0.14 | 1.00 | -0.05 | -0.13 |
| Money Growth | | | 1.00 | 0.50 | 0.06 | 0.15 | -0.99 | 0.06 | 0.13 |
| Inflation | | | | 1.00 | -0.06 | -0.12 | -0.50 | 0.23 | -0.15 |
| % Peso Debt | | | | | 1.00 | -0.21 | -0.06 | 0.01 | -0.02 |
| Debt/GDP | | | | | | 1.00 | -0.14 | -0.81 | 0.98 |
| Peso IR Spread | | | | | | | 1.00 | -0.04 | -0.13 |
| $ IR Spread | | | | | | | | 1.00 | -0.83 |
| Default | | | | | | | | | 1.00 |

Note: Data from the simulated model were collected, and relevant model variable correlations were calculated. The table above presents results.

## Table 4: Mexican Data Correlations

| | $A$ | GDP Growth | Debt/GDP | M1 Growth | Inflation | % Peso Debt | For. Curr. IR Spread |
|---|---|---|---|---|---|---|---|
| $A$ | 1.00 | 0.13 | 0.06 | -0.64 | -0.72 | 0.88 | -0.16 |
| GDP Growth | | 1.00 | -0.29 | 0.18 | -0.20 | -0.18 | -0.37 |
| Debt/GDP | | | 1.00 | 0.12 | -0.26 | 0.24 | -0.04 |
| M1 Growth | | | | 1.00 | 0.58 | -0.37 | 0.01 |
| Inflation | | | | | 1.00 | -0.54 | 0.16 |
| % Peso Debt | | | | | | 1.00 | -0.36 |
| For. Curr. IR Spread | | | | | | | 1.00 |

Note: This table shows correlations in relevant variables using observed Mexican data from 1980 through 2010. Sources: Reinhart and Rogoff (2009), the World Bank Database, BIS Sovereign Debt Tables, Jeanne and Guscina (2006), and Cowan et al. (2006).

## Table 5: Simulations by Productivity Level

| $A$ | % Peso Debt | GDP | Debt/GDP | Price/Money | Money Growth | Expected Inflation | Default Freq. | q | q* |
|---|---|---|---|---|---|---|---|---|---|
| 0.21 | 50.10 % | 2.55 | 0.81 | 11.96 | 0.00 % | -26.22 % | 50.00 % | 1.33 | 0.29 |
| 0.23 | 33.75 % | 2.61 | 0.81 | 10.52 | -3.74 % | -22.85 % | 33.33 % | 1.27 | 0.52 |
| 0.24 | 33.78 % | 2.66 | 0.80 | 9.64 | -4.58 % | -20.59 % | 33.33 % | 1.24 | 0.52 |
| 0.25 | 33.81 % | 2.71 | 0.79 | 8.93 | -5.36 % | -18.62 % | 33.33 % | 1.21 | 0.52 |
| 0.26 | 33.91 % | 2.75 | 0.77 | 7.87 | -8.59 % | -16.82 % | 33.33 % | 1.18 | 0.52 |
| 0.27 | 1.41 % | 2.78 | 0.35 | 9.53 | 0.00 % | -13.90 % | 0.00 % | 1.14 | 0.98 |
| 0.28 | 0.52 % | 2.83 | 0.34 | 6.58 | -10.80 % | -14.25 % | 0.00 % | 1.15 | 0.98 |
| 0.29 | 0.56 % | 2.86 | 0.33 | 6.17 | -11.64 % | -12.80 % | 0.00 % | 1.13 | 0.98 |
| 0.31 | 0.59 % | 2.90 | 0.33 | 5.80 | -12.42 % | -11.41 % | 0.00 % | 1.11 | 0.98 |
| 0.32 | 0.63 % | 2.93 | 0.33 | 5.46 | -13.16 % | -10.06 % | 0.00 % | 1.09 | 0.98 |
| 0.33 | 0.67 % | 2.96 | 0.32 | 5.15 | -13.87 % | -8.73 % | 0.00 % | 1.08 | 0.98 |
| 0.34 | 0.70 % | 3.00 | 0.32 | 4.86 | -14.55 % | -7.43 % | 0.00 % | 1.06 | 0.98 |
| 0.35 | 0.75 % | 3.03 | 0.31 | 4.58 | -15.22 % | -6.11 % | 0.00 % | 1.05 | 0.98 |
| 0.36 | 0.79 % | 3.07 | 0.31 | 4.32 | -15.90 % | -4.77 % | 0.00 % | 1.03 | 0.98 |
| 0.37 | 0.84 % | 3.11 | 0.31 | 4.05 | -16.59 % | -3.35 % | 0.00 % | 1.02 | 0.98 |
| 0.39 | 0.90 % | 3.15 | 0.01 | 3.77 | -17.36 % | -1.75 % | 0.00 % | 1.00 | 0.98 |

Note: This table shows simulation results when the economy receives the same productivity shock every period. These results present how policies and model variables vary by productivity draw.

## Table 6: Model Comparison: Effect of Debt Instruments

| Model | Welfare Diff. (CM Cons.) | Debt/GDP | Inflation | Velocity | Markup | Default Freq. | % Peso Debt |
|---|---|---|---|---|---|---|---|
| *No Commitment* | | | | | | | |
| No Debt | - | - | -12.20 % | 19.12 | 24.33 % | - | - |
| Peso Debt | 0.03 % | 0.01 | -12.20 % | 19.12 | 24.33 % | 0.00 % | - |
| Dollar Debt | 6.75 % | 0.31 | -12.20 % | 19.12 | 24.33 % | 2.55 % | - |
| Both Debts | 6.76 % | 0.33 | -10.71 % | 20.52 | 24.66 % | 2.56 % | 2.94 % |
| *Commit to $\bar{g}$* | | | | | | | |
| No Debt (commit $\bar{g}$) | 0.00 % | - | -12.20 % | 19.12 | 24.33 % | - | - |
| Peso Only (commit $\bar{g}$) | 0.18 % | 0.02 | -12.20 % | 19.12 | 24.33 % | 0.00 % | - |
| Dollar Only (commit $\bar{g}$) | 6.75 % | 0.31 | -12.20 % | 19.12 | 24.33 % | 2.55 % | - |
| Both Debts (commit $\bar{g}$) | 7.15 % | 0.34 | -12.20 % | 19.12 | 24.33 % | 2.56 % | 6.16 % |

Note: This table shows the implications of gaining access to different debt instruments. Starting from a no-debt regime, I present relevant statistics as the government gains access to first peso-denominated debt only, then dollar-denominated debt only, and lastly both types of debt. Welfare is stated in terms of annual CM consumption and is compared to the no-debt regime benchmark. In the second half of the table, I present the same statistics when the government is forced to commit to a monetary policy rule such that $g_t = \bar{g}$.

## Table 7: Model Comparison: Effect of Monetary Policy Windows Around $\bar{g}$

| Model | Welfare Diff. (CM Cons.) | Debt/GDP | Inflation | Velocity | Markup | Default Freq. | % Peso Debt |
|---|---|---|---|---|---|---|---|
| $\bar{g}$ | 0.05 % | 0.34 | -12.20 % | 19.12 | 24.33 % | 2.56 % | 6.16 % |
| $\bar{g} \pm 5\%$ | 0.04 % | 0.34 | -11.96 % | 19.31 | 24.33 % | 2.56 % | 5.82 % |
| $\bar{g} \pm 10\%$ | 0.03 % | 0.34 | -11.24 % | 19.58 | 24.33 % | 2.56 % | 5.18 % |
| $\bar{g} \pm 25\%$ | 0.01 % | 0.34 | -11.03 % | 20.09 | 24.33 % | 2.56 % | 4.32 % |
| Full Money Choice | - | 0.33 | -10.71 % | 20.52 | 24.33 % | 2.56 % | 2.94 % |

Note: This table shows model results at different levels of monetary policy commitment. In each simulation, the government has monetary policy discretion in some window around the growth rate $\bar{g}$. Each variation changes the size of this window.

Table 8: Model Comparison: Effect of Monetary Policy Commitment Choices

| Model | Welfare Diff. (CM Cons.) | Debt/GDP | Inflation | Price Variance | Velocity | Markup | Default Freq. | % Peso Debt |
|---|---|---|---|---|---|---|---|---|
| $\bar{g} - 20\%$ | 0.02 % | 0.35 | -29.73 % | 5.85 | 19.11 | 24.33 % | 2.56 % | 7.68 % |
| $\bar{g} - 10\%$ | 0.02 % | 0.35 | -20.94 % | 5.86 | 19.11 | 24.33 % | 2.56 % | 7.04 % |
| $\bar{g} - 5\%$ | 0.01 % | 0.34 | -13.65 % | 5.90 | 19.12 | 24.33 % | 2.56 % | 6.42 % |
| $\bar{g}$ | - | 0.34 | -12.20 % | 6.05 | 19.12 | 24.33 % | 2.56 % | 6.16 % |
| $\bar{g} + 5\%$ | -0.01 % | 0.33 | -11.51 % | 6.24 | 20.07 | 24.76 % | 2.56 % | 6.11 % |
| $\bar{g} + 10\%$ | -0.02 % | 0.33 | -10.33 % | 6.61 | 21.00 | 25.18 % | 2.56 % | 6.02 % |
| $\bar{g} + 20\%$ | -0.04 % | 0.33 | -8.92 % | 7.34 | 22.89 | 25.94 % | 2.56 % | 5.96 % |

Note: This table shows model results under full monetary policy committment to different rates of money growth. In each simulation, the government commits fully to a growth rate of money at or around the "efficient" growth rate $\bar{g}$.