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Bequests, Inter Vivos Transfers, and Wealth Distribution

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Abstract

This paper extends the heterogeneous agent overlapping generations model with bequests in Nishiyama (2000) by adding two-way intergenerational altruism and inter vivos transfers. Calibrating the model to the U.S. economy, the paper measures time preference and intergenerational altruism consistent with the economy's capital-output ratio and the sizes of intergenerational transfers. In the model, households in the same dynasty play a Nash game in each period to determine their optimal consumption, working hours, inter vivos transfers, and savings. The model suggests that when deciding the level of bequests, a parent household considers the future utility of its child households, on average, about 20 percent less than it considers its own future utility. But, the parent household's motive for inter vivos transfers is much weaker than its motive for altruistic bequests. The model replicates the wealth distribution of the United States fairly well in terms of the Gini coefficient although the top 1 percent of the population holds proportionately less wealth than is observed in the data. The paper also analyzes the effect of intergenerational transfers on wealth accumulation and distribution.

1 Introduction

Macroeconomic analyses usually rely on either an infinite horizon model or an overlapping generations model. Those analyses implicitly assume a household is either perfectly altruistic or completely selfish. But, when economists evaluate fiscal policies that involve income redistribution between generations, the policy implication depends critically on the extent to which the Ricardian Equivalence proposition holds. In other words, we need to know to what extent households within a dynasty are altruistic toward each other and how those households react when the government introduces a new policy.

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Table 1: The Features of the Two Models

	Nishiyama (2000)	Present Paper
Intergenerational Altruism	One-Sided	Two-Way
Lifetime Uncertainty	3 or 4 Periods	3 or 4 Periods
Fertility Shock	None	0 or n Children
Bequests	Altruistic and Accidental	Altruistic and Accidental
Inter Vivos Transfers	None	Two-Way (Altruistic)

In my previous paper (Nishiyama, 2000), I developed a heterogeneous agent overlapping generations model with altruistic and accidental bequests, and I measured time preference and parental altruism in the U.S. economy. That model considered both parental altruism and lifetime uncertainty, and it captured the strategic behavior between a parent household and its child households by assuming a Nash game. The result of the calibration showed that a parent household, on average, cares about its adult child households roughly 30 percent less than it cares about itself.[1]

However, several panel data sets (e.g., Gale and Scholz, 1994) show a significant number of inter vivos transfers between a parent household and its child households, which the previous model did not consider. This simplification is justified if the capital market is perfect. But, if a borrowing constraint exists, then it is also beneficial for an altruistic parent household to make transfers before it dies. Also, the possibility of adult child households making gifts to their parent household affects both of their life-cycle saving schedules.

If we relax the model to allow households to make inter vivos transfers in each period, how will the wealth accumulation and distribution in the United States change? This paper extends the heterogeneous agent overlapping generations model by adding two-way intergenerational altruism, lifetime uncertainty, a fertility shock, and borrowing constraints, and it measures time preference and intergenerational altruism through the calibration of the model to the U.S. economy (see Table 1). The paper also analyzes, based on the obtained parameters, the effect of bequests and gifts on wealth accumulation and inequality.

One of the features of this extended model is that it involves both an infinite horizon economy (with borrowing constraints) and a pure life-cycle economy as two opposite cases. It is likely that the economy has imperfectly altruistic households, and it can be shown as an economy located between those two extremes.

Similarly to the previous paper, the main parameters — time preference and two-way intergenerational altruism — are obtained simultaneously. This is done through the calibration of the model so that the steady-state equilibrium is consistent with the key statistics observed in the United States: the capital-output ratio and the relative sizes of bequests and inter vivos transfers. For the steady-state economy to be consistent, a parent household would have to consider the future utility of its adult child households about 20 percent less than it considers its own future utility.[2] But, the parent household is actually less willing to make inter vivos

[1] In that calibration, I included a half of inter vivos transfers to children or grandchildren in bequests from parents to children as disguised bequests to avoid estate taxes.

[2] The discount rate is lower in this paper than in the previous one (20 percent instead of 30 percent), partly

transfers to its adult child households, discounting the current utility of its adult children by more than 50 percent.[3]

The model also replicates the wealth distribution of the United States fairly well. The Gini coefficient of wealth distribution of the baseline economy turns out to be 0.701.[4] But, the share of wealth of the top 1 percent of households is 14.6 percent in the model, still lower than the 29.6 percent in the data.[5] The effects of bequests and inter vivos transfers on wealth distribution are not very large. Under the parameter setting in this paper, those transfers, in total, increase the inequality in a closed economy, but decrease it in a small open economy.

The rest of this paper is laid out as follows: Section 2 discusses previous literature about bequests and inter vivos transfers, Section 3 describes the economy and the extended model, and Section 4 shows the calibration of the model and the obtained main parameters. Section 5 uses policy experiments to examine the effects of altruistic and accidental bequests as well as inter vivos transfers on wealth accumulation and inequality, and Section 6 concludes the paper. The appendixes to the paper explain the algorithm of computing household decision rules and the welfare measures.

2 Previous Literature

In this paper, I construct an altruistic model of bequests and inter vivos transfers based on the strategic behavior between a parent household and its adult child households and measure the degrees of altruism between those households. To my knowledge, except for Nishiyama (2000), few analyses try to measure the degree of intergenerational altruism using a dynamic general equilibrium model.

Lord and Rangazas (1991) used a partial equilibrium model to evaluate the effect of bequests on wealth accumulation by explicitly assuming their altruistic parameter to be unity, i.e., parents care about their descendants as much as they care about themselves. Fuster, İmrohoroğlu, and İmrohoroğlu (1999) introduced lifetime uncertainty and inter vivos transfers as well as bequests to their dynamic general equilibrium model, but they assumed perfect risk sharing between parents and children. De Nardi (1999) introduced a "warm glow" bequest motive to her dynamic general equilibrium model and chose the parameters to create a reasonable size of bequests. But, since her model does not consider the state of recipients (child households), it cannot measure the degree of parental altruism exactly.

The Share of Transfer Wealth. Many other papers have tried to measure the shares of life-cycle wealth and transfer wealth. On the one hand, Kotlikoff and Summers (1981) constructed a measure of transfer wealth and showed that it accounts for at least 80 percent of total wealth in the United States. On the other hand, Modigliani (1988) and others estimated

because I distinguish the elderly households with and without children in this paper.

[3]The parent household may behave as if it is less altruistic when its adult child households are relatively young, so that the parent will not discourage the children from working. I will discuss this result more in Section 6.

[4]For example, according to the 1989 wealth data from the Panel Study of Income Dynamics (PSID), the wealth Gini coefficient of married households is 0.71.

[5]See Díaz-Giménez et al. (1997), Table 6.

that the share of bequeathed wealth is at most 20 percent.[6] Gale and Scholz (1994) examined inter vivos transfers, college expenditures, and trusts and life insurance purchases, and they concluded that intentional transfers account for at least 20 percent of total wealth and more than 50 percent if bequests are included.

Those estimates have two limitations, however: first, the transfer wealth measure is sensitive to the assumption of a steady-state interest rate and growth rate; second, even if we know the exact share of transfer wealth, we still cannot estimate to what extent national wealth would be reduced if there were no intergenerational transfers. For example, if a 100 percent estate tax were introduced to eliminate all bequests, forward-looking child households would increase their life-cycle savings because they would not expect bequests from their parents.[7]

Intergenerational Transfer Motives. Other papers have examined to what extent bequests are intentional or accidental, and if they are intentional whether bequests and inter vivos transfers are altruistic, selfish, or strategic. Hurd (1987) compared the dissaving pattern of old households with children with that of households without children and concluded that bequests are mostly accidental. Wilhelm (1996) showed that parents tend to leave equal bequests to each of their children even if the children's earnings differ significantly, and he concluded that bequests are not altruistic.

In contrast, Menchik and David (1983) showed that elderly households do not dissave and concluded that bequests are intentional. Bernheim (1991) used the same data set as Hurd, the Longitudinal Retirement Household Survey, and concluded that bequests are intentional. According to Bernheim's paper, households adjust their level of bequeathable wealth by reducing their private annuity or increasing their life insurance when their public pension increases.

Altruistic Hypotheses. This paper considers both intentional transfers (bequests and inter vivos transfers) and accidental bequests due to lifetime uncertainty. Regarding the question of whether the intentional transfers are altruistic, selfish, or strategic, this paper assumes that intentional transfers are motivated by altruism, although a part of bequests and gifts may be selfish or strategic.[8]

One of the main criticisms of the altruistic bequest model is that bequests are in many cases divided equally by parents even if the earnings of their children differ significantly. According to Wilhelm (1996), 76.6 percent of parents divided their estates almost equally (within ± 2 percent). This does not contradict the altruistic model, however, if we consider the psychic cost of making unequal bequests. Stark (1998) introduced the notion of the "relative deprivation" in children's utility function to show that the equal division of bequests and the altruistic bequest model are not mutually exclusive.

[6] See Modigliani (1988) for several estimates of others. The difference between those two conclusions is due in part to different definitions of transfer wealth and bequeathed wealth.

[7] To what extent the savings of child households would change depends partly on how the government would use the revenue from a 100 percent estate tax on accidental bequests.

[8] Selfish bequests are sometimes called "joy-of-giving" bequests or "bequests-as-consumption." Also, "warm glow" bequests belong to that category. Strategic bequests are sometimes called "gift-exchange" bequests. Inter vivos transfers may be due to risk-sharing arrangements between households.

Another criticism of altruistic hypotheses is that inter vivos transfers from parents to children do not completely compensate for the income changes of parents and children. Altonji, Hayashi, and Kotlikoff (1997) showed that parents increase transfers by only 13 cents when their income increases by one dollar and that of their children decreases by one dollar. But, in the presence of asymmetric information about children's working ability and efforts, it may be optimal for parents to offer the partial insurance on children's income shocks to avoid moral hazard. In fact, empirical analyses by Altonji et al. (1997) and Wilhelm (1996) show that inter vivos transfers and bequests, respectively, are decreasing in the recipient's income and imply that intergenerational transfers are at least partially motivated by altruism.

Implicit Insurance Contracts. Intergenerational transfers may be motivated by the income shock and lifetime uncertainty of households in the absence of perfect insurance and annuity markets. Even if parents and children are not altruistic toward each other, it is beneficial for them to make a risk-sharing contract if they can avoid enforceability problems and adverse selection. But, if parents and children are not altruistic at all, the sum of insurance payments should be close to that of insurance benefits. According to Gale and Scholz (1994), inter vivos transfers from parents to children are about ten times larger than those from children to parents; if we consider other transfers — such as bequests, trusts, and life insurance — that difference becomes much larger.

Those net transfers from parents to children are not explained solely by the risk-sharing motive. Also, for the implicit annuity contract between parents and children, it is enough to distinguish accidental bequests from other intentional transfers. This is because the price of the annuity that parents have to pay is, on average, not very different from the amount of accidental bequests.

Strategic Bequest Motive. Parents may want to keep their wealth in a bequeathable form, even in the presence of perfect annuity markets, to attract their children's attention (e.g., Bernheim, Shleifer, and Summers, 1985). In that case, parents' bequests and other transfers to their children are the payments for their children's services, such as telephone calls and visits.[9] But, as I mentioned before, Wilhelm (1996) showed that a majority of bequests are divided equally. Also, Behrman and Rosenzweig (1998) showed that the relationship between the amount of bequests and the number of visits across children is not significant and rejected the framework in which parents use threats of disinheritance to elicit more visits from their children.

3 Model

This section describes a four-period heterogeneous agent overlapping generations model with bequests and inter vivos transfers. The model differs from the previous one (Nishiyama, 2000) in the following three ways. First, child households are also altruistic toward their parent household; second, both a parent household and its child households make inter vivos

[9] Strategic bequests and altruistic bequests are not mutually exclusive because parents may value telephone calls and visits by children only when the parents are altruistic.

transfers based on their degrees of altruism and an estate and gift tax; and third, a household receives a fertility shock, i.e., some households have n child households and others have none. Similarly to the previous model, this model considers both altruistic and accidental bequests. Also, households in the same dynasty behave strategically to determine their consumption, working hours, gifts, and savings.

3.1 Two-Way Intergenerational Altruism

Before moving on to describe the model, I first want to show how to measure the degree of intergenerational altruism, using a simple two-period overlapping generations economy without uncertainty and population growth.

When there is no altruism, the lifetime utility of a generation g household, which lives two periods, is shown as

$$u^g = u\,(c_1^g) + \beta\, u(c_2^g), \tag{1}$$

where $u(c_i^g)$ denotes instantaneous utility from the consumption at age i, and β is a usual time preference parameter of this household. Suppose that a child household (a generation $g+1$ household) is born to the household at the beginning of age 2 and that a household is altruistic toward its descendants with a discount factor η per generation. Then the total utility of a household of generation g is defined as the infinite sum of the lifetime utility of each generation, i.e.,

$$\begin{aligned} \tilde{U}_1^g &= u^g + (\beta\,\eta)\; u^{g+1} + (\beta\,\eta)^2\, u^{g+2} + (\beta\,\eta)^3\, u^{g+3} + ... \\ &= u^g + (\beta\,\eta)\; \tilde{U}_1^{g+1}, \end{aligned}$$

where $\eta \geq 0$ and $\beta\,\eta < 1$. Using (1), we can arrange this total utility as the infinite sum of instantaneous utility of the dynasty, which is a combination of a parent household and its child household, i.e.,

$$\tilde{U}_1^g = u\,(c_1^g) + \beta \sum_{i=0}^{\infty} (\beta\,\eta)^i \left\{ u(c_2^{g+i}) + \eta\, u\left(c_1^{g+i+1}\right)\right\}.$$

The total utility of a generation g household of age 2 is also shown as

$$\tilde{U}_2^g = \sum_{i=0}^{\infty} (\beta\,\eta)^i \left\{ u(c_2^{g+i}) + \eta\, u\left(c_1^{g+i+1}\right)\right\}.$$

In this paper, η is the degree of parental altruism because it shows the relative importance of the adult child's utility to its parent household.

Suppose that the degree of parental altruism differs when the parent is deceased η and when the parent is alive η_0. The total utility of an age 2 parent household, U_2^g, becomes

$$U_2^g = \left\{ u(c_2^g) + \eta_0\, u\left(c_1^{g+1}\right)\right\} + \beta\,\eta\, U_2^{g+1}. \tag{2}$$

In other words, η_0 represents the parent's inter vivos transfer motive and η represents its bequest motive.

Suppose that the child household is also altruistic toward its parent household with a discount factor ρ. I call ρ the degree of the child's altruism toward its parent household. The total utility of this age 1 child household is written as

$$U_1^{g+1} = \left\{\rho\, u\left(c_2^g\right) + u\left(c_1^{g+1}\right)\right\} + \beta\, U_2^{g+1},$$

where the regularity condition is $\rho \geq 0$, $\eta \geq 0$, $\eta_0 \geq 0$, $\rho\,\eta_0 \leq 1$, and $\beta\,\eta < 1$.

Notice that the discount factor of the parent household on its child's utility from age 1 to age 2, $\beta\,\eta/\eta_0$, is different from the child's own discount factor β. Still, equation (2) shows that this is a time consistent problem.[10]

When $\rho\,\eta_0 = 1$ and $\eta_0 = \eta$, we have $U_2^g = \eta\, U_1^{g+1}$, and this model becomes an infinite horizon model (a dynasty model) as long as inter vivos transfers are allowed. But, when $\rho = \eta = \eta_0 = 0$, this model becomes an overlapping generations model (a pure life-cycle model) in which households are completely selfish.

In this paper, I do not use the model with "two-sided" altruism, in which the total utility of a household can be nested toward both its ascendant side and its descendant side, to avoid the complexity from the "hall of mirror" effects.[11] In other words, a household does not care about its ascendants that have already died.

This simplification is justified for the following reasons: First, in this setting, the model involves both an infinite horizon model and a pure life-cycle model as two polar cases, and this is enough to analyze the Ricardian Equivalence proposition. Second, in Section 4, I will show that the parameter ρ is very small, and the difference between the results from this model and a "two-sided" altruism model is negligible.

3.2 Economy

The model is based on a standard growth economy that consists of a large number of households, a perfectly competitive firm, and a government. Each household is assumed to act as a single person.[12]

In each period, new households are born without any wealth. The life span of each household is either three or four periods. One period in this model corresponds to 15 years starting from the actual age of 30. A household dies either at the end of age 3 or at the end of age 4. When a household reaches age 3, its n child households of age 1 are "born" with probability ξ, and the former becomes a parent household (See Figure 1).

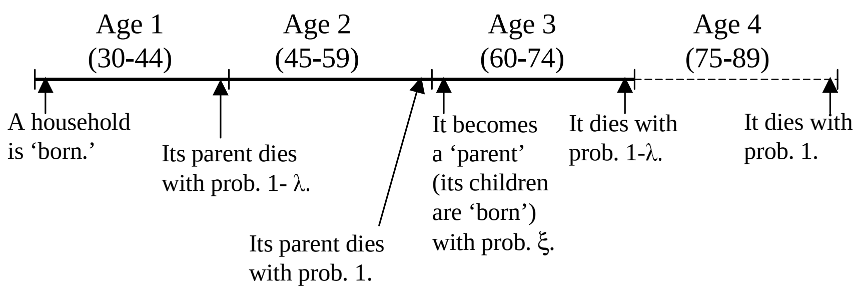
When a household is age 1 or 2, its working ability (labor productivity) at each age is stochastically determined. It receives labor income (earnings) according to the market wage rate, its working hours, and its working ability. A household of age 3 or 4 is assumed to

[10] The total utility of a generation g parent household is $U_2^g = \sum_{s=g}^{\infty} (\beta\eta)^{s-g} \left\{u(c_2^s) + \eta_0\, u\left(c_1^{s+1}\right)\right\}$, and the total utility of its child household (generation $g+1$) is $U_1^{g+1} = \left\{\rho\, u\left(c_2^g\right) + u\left(c_1^{g+1}\right)\right\} + \beta \sum_{s=g+1}^{\infty} (\beta\eta)^{s-g-1} \left\{u(c_2^s) + \eta_0\, u\left(c_1^{s+1}\right)\right\}$. When $\eta_0 \neq \eta$, there is a conflict of interests between the parent household and its child household about the child's consumption. Still, this model can be solved as a time consistent problem.

[11] Here, I use the term "two-way" altruism to distinguish it from "two-sided" altruism. For "two-sided" altruism, see Kimball (1987).

[12] In the calibration of the model, a household is assumed to be a married couple, but their decisions are made jointly. Also, there is assumed to be no strategic interaction between siblings.

Figure 1: The Life Cycle of a Household



be retired. Though it can work at home to produce a limited amount of consumption goods and services, its working ability is assumed to be low and deterministic. A household can hold only one kind of assets. It receives capital income according to its wealth level and the market interest rate. The wealth of each household must be nonnegative.

A household pays federal income tax according to its total income. A household that inherits any wealth from its parent also pays federal and state estate taxes.[13] In addition, a household of age 1 or 2 pays payroll tax for Social Security and Medicare. A household of age 3 or 4 receives Social Security benefits. The Social Security system is assumed to be the defined benefit type and, for simplicity, the size of the benefit is assumed to be the same for all households.

At any time, there are two types of dynasties — the dynasties with both a parent household and its child households (Type I), and the dynasties without any overlapping generations (Type II). Figure 2 shows two types of dynasties in this economy. Every parent household is assumed to be equally altruistic and cares about its child households. In Type I dynasties, both a parent household and its child households are altruistic, but to a different extent.

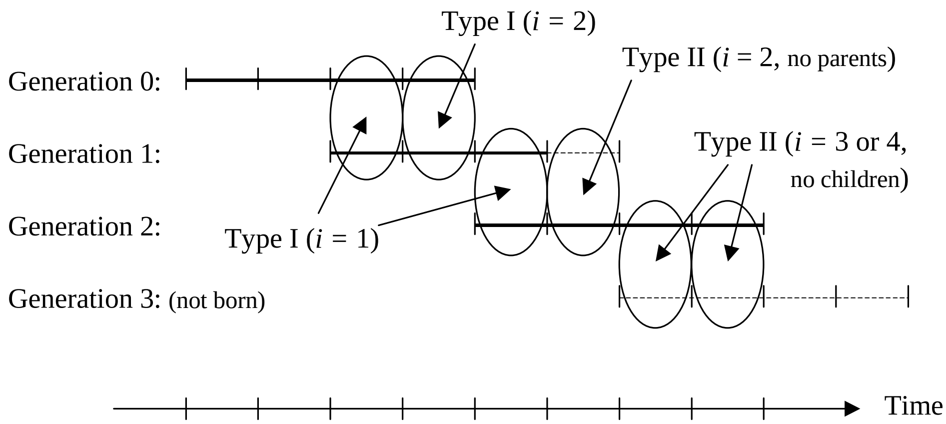
Beginning-of-period wealth of a parent household and its child households, and the working ability of the child households, are known to each other. A parent and its children choose, simultaneously, their own optimal consumption, working hours, gifts (to each other), and end-of-period wealth.

3.3 Households' Problem

For Type I dynasties, the state of each dynasty is shown by the ages of a parent household and its child households $\{(3,1), (4,2)\}$, the beginning-of-period wealth of the parent $a_p \in A = [0, a_{\max}]$ and that of its children $a_k \in A$, and the labor productivity (which determines hourly wage) of the children $e_k \in E = [E_{\min}, E_{\max}]$. For Type II dynasties, the state of each dynasty is shown simply by the age of a household $\{2, 3, 4\}$, the beginning-of-period

[13]In the baseline economy, the gift tax on inter vivos transfers is not considered and the tax rate is assumed to be 0 percent. A couple (two parents) can avoid the federal gift tax on gifts of up to \$20,000 per child per year.

Figure 2: Two Types of Dynasties



wealth $a \in A$, and the working ability $e \in E$. But, for the households of age 3 or 4, since their working ability is assumed to be deterministic, their states are shown as $(a, -)$ instead of (a, e). In the calibration, the working ability of age i, $e_{k,i}$ or e_i, is a member of $\{e_i^1, e_i^2, e_i^3\}$ for households of age 1 or 2, and it follows a Markov process.

For notational simplicity, let $\mathbf{s}_I$ and $\mathbf{s}_{II}$ denote the states of a Type I dynasty and a Type II dynasty, respectively, where

$$\mathbf{s}_I = (a_p, a_k, e_k)\,,\ \mathbf{s}_{II} = (a, e)\ \text{ or }\ (a, -)\,.$$

Then the value function of a Type I household of age i is denoted as $v_{I,i}(\mathbf{s}_I)$, and that of a Type II household of age i is denoted as $v_{II,i}(\mathbf{s}_{II})$.

3.3.1 Type I Households

An Age 3 Parent and Its Age 1 Children. Let c_p, h_p, g_p, and a'_p denote the parent household's consumption, working hours, inter vivos gifts to its child households, and the end-of-period wealth level, respectively. Similarly, let c_k, h_k, g_k, and a'_k denote each of its child household's consumption, working hours, inter vivos gifts to the parent household, and the end-of-period wealth level, respectively. Also, let $\mathbf{\Phi}_k$ denote the parent household's conjecture about its child households' decision, n denote the number of child households, λ denote the survival rate at the end of age 3, r denote the rate of return on the capital, w denote the wage rate per efficient unit of labor, μ denote the growth rate of the economy, $\tau_F(.)$ be a federal income tax function, $\tau_S(.)$ be a payroll tax function for Social Security and Medicare, $\tau_E(.)$ be a federal and state estate and gift tax function, and tr_{SS} denote Social Security benefits.

The value function of an age 3 parent household is shown as

$$\begin{aligned} v_{I,3}(\mathbf{s}_I;\mathbf{\Phi}_k) &= \max_{c_p,h_p,g_p,a'_p} \Big\{ u(c_p,h_p) + \eta_0\, n\, u(c_k,h_k) \\ &\quad + \beta\, E\left[\lambda\, v_{I,4}(\mathbf{s}'_I) + (1-\lambda)\, \eta\, n\, v_{II,2}(\mathbf{s}'_{II}) \,|\, e_k \right] \Big\} \end{aligned} \tag{3}$$

subject to

$$\begin{aligned} a'_p &= \frac{1}{1+\mu} \{ w\, e_p h_p + (1+r)\, a_p + tr_{SS} - \tau_F(r\, a_p) - c_p \\ &\quad - (g_p - n\, g_k) \} \geq 0, \end{aligned} \tag{4}$$

where $\mathbf{s}_I$ is the state of this dynasty,

$$\mathbf{s}_I = (a_p, 0, e_k)\,,$$

$\mathbf{\Phi}_k$ is the parent's conjecture of its child households' decision,

$$\mathbf{\Phi}_k = (c_k, h_k, g_k, a'_k),$$

and the law of motion of the state of this dynasty is

$$\begin{aligned} \mathbf{s}'_I &= \left(a'_p, a'_k, e'_k \right), \\ \mathbf{s}'_{II} &= \left(a'_k + a'_p/n - \tau_E\left(a'_p/n \right), e' \right). \end{aligned} \tag{5}$$

The parent household chooses its optimal consumption c_p, working hours (housework only) h_p, inter vivos transfers g_p, and end-of-period wealth level (normalized by the economic growth) a'_p, taking the decision of its child households $\mathbf{\Phi}_k$ as given. It discounts the utility of each of n child households by η_0 when it is alive. At the end of age 3, the parent household dies with probability $1-\lambda$. The value of this household at the beginning of the next period is the weighted average of its own future value $v_{I,4}$ (when this household is alive) and its n children's future value $n\, v_{II,2}$ discounted by η (when this household is deceased).[14] The term $E\,[\,.\,|\, e_k]$ denotes a conditional expectation given that the current working ability of an age 1 child household is e_k, i.e.,

$$E\left[v_{I,4}(\mathbf{s}'_I) \,|\, e_k \right] = \int_E v_{I,4}(\mathbf{s}'_I)\, \pi_{1,2}(e'_k \,|\, e_k)\, \mathrm{d}e'_k,$$

where $\pi_{1,2}(e'_k \,|\, e_k)$ is a conditional probability of the working ability being e'_k in the next period. The equation (4) is a budget constraint of this parent household, where $g_p - n\, g_k$ denotes the net gifts given to its child households. In the baseline economy, gift tax is not considered. When the parent household dies, its end-of-period wealth a'_p is split equally and bequeathed to each of n child households.

[14] In this model, I assume that the utility the parent household receives from its child households is linear in the number of child households.

Let $\mathbf{\Phi}_p$ denote the child household's conjecture about its parent household's decision. The value function of an age 1 child household is shown as

$$\begin{aligned} v_{I,1}(\mathbf{s}_I;\mathbf{\Phi}_p) &= \max_{c_k,h_k,g_k,a'_k} \Big\{ u(c_k,h_k) + \rho\, n^{-1} u\left(c_p,h_p\right) \\ &\quad + \beta\, E\left[\lambda\, v_{I,2}(\mathbf{s}'_I) + (1-\lambda)\, v_{II,2}(\mathbf{s}'_{II}) \mid e_k\right] \Big\} \end{aligned} \tag{6}$$

subject to

$$\begin{aligned} a'_k &= \frac{1}{1+\mu}\{w\, e_k h_k + (1+r)\, a_k - \tau_F(w\, e_k h_k + r\, a_k) - \tau_S(w\, e_k h_k) - c_k \\ &\quad - (g_k - g_p/n)\} \geq 0, \end{aligned} \tag{7}$$

where $\mathbf{\Phi}_p$ is the child's conjecture of its parent household's decision,

$$\mathbf{\Phi}_p = (c_p, h_p, g_p, a'_p),$$

and the law of motion of the state is (5).

The child household chooses its optimal consumption c_k, working hours h_k, inter vivos transfers g_k, and end-of-period wealth level a'_k, taking the decision of its parent household $\mathbf{\Phi}_p$ as given. It discounts the utility of its parent household by ρ. The value of the child household at the beginning of the next period is the weighted average of its own future value when its parent is alive, $v_{I,2}$, and its future value when its parent is deceased, $v_{II,2}$. The equation (7) is a budget constraint of this child household, and $g_k - g_p/n$ denotes the net gifts given to its parent household.

Let $\mathbf{d}_p$ and $\mathbf{d}_k$ be the set of decisions of a parent household and each of its child households, respectively, i.e.,

$$\mathbf{d}_p = (c_p, h_p, g_p, a'_p), \quad \mathbf{d}_k = (c_k, h_k, g_k, a'_k).$$

Solving equations,

$$\mathbf{R}_3\left(\mathbf{d}_k;\mathbf{s}_I\right) = \mathbf{d}_p, \quad \mathbf{R}_1\left(\mathbf{d}_p;\mathbf{s}_I\right) = \mathbf{d}_k,$$

where $\mathbf{R}_3\left(\mathbf{d}_k;\mathbf{s}_I\right)$ and $\mathbf{R}_1\left(\mathbf{d}_p;\mathbf{s}_I\right)$ are the best response functions of a parent and its children, respectively, Nash equilibrium decision rules are obtained as

$$\mathbf{d}_{I,i}(\mathbf{s}_I) = \Big(c_{I,i}(\mathbf{s}_I), h_{I,i}(\mathbf{s}_I), g_{I,i}(\mathbf{s}_I), a'_{I,i}(\mathbf{s}_I)\Big)$$

for $\mathbf{s}_I \in A^2 \times E$, where $i = 3$ or 1.

An Age 4 Parent and Its Age 2 Children. An age 4 parent household is assumed to die at the end of this period, and its child households become parent households with probability ξ at the beginning of the next period. So, the value function of an age 4 parent household is shown as

$$\begin{aligned} v_{I,4}(\mathbf{s}_I;\mathbf{\Phi}_k) &= \max_{c_p,h_p,g_p,a'_p} \Big\{ u(c_p,h_p) + \eta_0\, n\, u(c_k,h_k) \\ &\quad + \beta\, \eta\, n\, E\left[\xi\, v_{I,3}(\mathbf{s}'_I) + (1-\xi)\; v_{II,3}\left(\mathbf{s}'_{II}\right) \mid e_k\right] \Big\} \end{aligned} \tag{8}$$

subject to (4), where the law of motion of the state is

$$\mathbf{s}'_I = (a'_k + a'_p/n - \tau_E(a'_p/n), 0, e'_k),$$

$$\mathbf{s}'_{II} = \left(a'_k + a'_p/n - \tau_E(a'_p/n), -\right). \tag{9}$$

The parent household considers its children's value at the beginning of the next period, which is the weighted average of $n\, v_{I,3}$ (when the child household becomes a parent) and $n\, v_{II,3}$ (otherwise), discounted by η.[15] Similarly, the value function of an age 2 child household is shown as

$$\begin{aligned} v_{I,2}(\mathbf{s}_I; \mathbf{\Phi}_p) \quad = \quad & \max_{c_k, h_k, g_k, a'_k} \Big\{ u(c_k, h_k) + \rho\, n^{-1} u\,(c_p, h_p) \\ & + \beta\, E\left[\xi\, v_{I,3}(\mathbf{s}'_I) + (1-\xi)\ v_{II,3}\,(\mathbf{s}'_{II})\ |\, e_k\right] \Big\} \end{aligned} \tag{10}$$

subject to (7), where the law of motion of the state is (9).

The decision rule of an age i household is obtained as

$$\mathbf{d}_{I,i}(\mathbf{s}_I) = \Big(c_{I,i}(\mathbf{s}_I), h_{I,i}(\mathbf{s}_I), g_{I,i}(\mathbf{s}_I), a'_{I,i}(\mathbf{s}_I)\Big)$$

for $\mathbf{s}_I \in A^2 \times E$, where $i = 4$ or 2.

3.3.2 Type II Households

The value function of an age 2 household without its parent household is simply

$$v_{II,2}(\mathbf{s}_{II}) = \max_{c,h,a'} \Big\{ u(c,h) + \beta\, E\left[\xi\, v_{I,3}(\mathbf{s}'_I) + (1-\xi)\ v_{II,3}\,(\mathbf{s}'_{II})\ |\, e\right] \Big\} \tag{11}$$

subject to

$$a' = \frac{1}{1+\mu}\left\{w\, e\, h + (1+r)\, a - \tau_F(w\, e\, h + r\, a) - \tau_S(w\, e\, h) - c\right\} \geq 0, \tag{12}$$

where the law of motion of the state is

$$\mathbf{s}'_I{=}(a', 0, e'_k), \quad \mathbf{s}'_{II}{=}(a', -).$$

The value function of an age 3 household without its child households is

$$v_{II,3}(\mathbf{s}_{II}) = \max_{c,h,a'} \Big\{ u(c,h) + \beta\, \lambda\, v_{II,4}\,(\mathbf{s}'_{II}) \Big\} \tag{13}$$

subject to

$$a' = \frac{1}{1+\mu}\left\{w\, e\, h + (1+r)\, a + tr_{SS} - \tau_F(r\, a) - c\right\} \geq 0, \tag{14}$$

where the law of motion of the state is

$$\mathbf{s}'_{II}{=}(a', -).$$

[15] To avoid introducing grandchildren into the model, I assume that a household does not know whether it will be a parent household until the beginning of age 3.

Finally, the value function of an age 4 household without its child households is

$$v_{II,4}(\mathbf{s}_{II}) = \max_{c,h,a'} u(c,h) \tag{15}$$

subject to (14).

The household chooses its optimal consumption c, working hours h, and end-of-period wealth level a'. The household's decision rules are obtained as

$$\mathbf{d}_{II,i}(\mathbf{s}_{II}) = \Big(c_{II,i}(\mathbf{s}_{II}), h_{II,i}(\mathbf{s}_{II}), a'_{II,i}(\mathbf{s}_{II})\Big)$$

for $\mathbf{s}_{II} \in A \times E$ and $i \in \{2,3,4\}$.

3.4 The Measure of Households

Let $x_{I,i}(\mathbf{s}_I)$ denote the measure of Type I households of age $i \in \{1,2\}$, and let $x_{II,i}(\mathbf{s}_{II})$ denote the measure of Type II households of age $i \in \{2,3,4\}$.[16] Also, let $X_{I,i}(\mathbf{s}_I)$ and $X_{II,i}(\mathbf{s}_{II})$ be the corresponding cumulative measures. The population of age 1 child households is normalized to be unity, i.e.,

$$\int_{A^2\times E} \mathrm{d}X_{I,1}(\mathbf{s}_I) = 1.$$

Let $\mathbf{1}_{[a'=y]}$ be an indicator function that returns 1 if $a' = y$ and 0 if $a' \neq y$. Then, the law of motion of the measure of Type I households is

$$x'_{I,2}(\mathbf{s}'_I) = \frac{\lambda}{1+\nu} \int_{A^2\times E} \mathbf{1}_{\left[a'_p = a'_{I,3}(\mathbf{s}_I)\right]} \mathbf{1}_{\left[a'_k = a'_{I,1}(\mathbf{s}_I)\right]} \pi_{1,2}(e'_k \mid e_k)\, \mathrm{d}X_{I,1}(\mathbf{s}_I), \tag{16}$$

and

$$\begin{aligned} x'_{I,1}(\mathbf{s}'_I) \quad &= \quad (1+\nu)\Big\{\int_{A^2\times E} \mathbf{1}_{\left[a'_p = a'_{I,2}(\mathbf{s}_I) + a'_{I,4}(\mathbf{s}_I)/n - \tau_E\left(a'_{I,4}(\mathbf{s}_I)/n\right)\right]} \\ &\qquad\qquad \times \pi_{2,1}(e'_k \mid e_k)\, \mathrm{d}X_{I,2}(\mathbf{s}_I) \\ &\quad + \int_{A\times E} \mathbf{1}_{\left[a'_p = a'_{II,2}(\mathbf{s}_{II})\right]} \pi_{2,1}(e'_k \mid e)\, \mathrm{d}X_{II,2}(\mathbf{s}_{II})\Big\}. \end{aligned} \tag{17}$$

The law of motion of the measure of Type II households is

$$\begin{aligned} x'_{II,2}(\mathbf{s}'_{II}) \quad &= \quad \frac{1-\lambda}{1+\nu}\Big\{\int_{A^2\times E} \mathbf{1}_{\left[a' = a'_{I,1}(\mathbf{s}_I) + a'_{I,3}(\mathbf{s}_I)/n - \tau_E\left(a'_{I,3}(\mathbf{s}_I)/n\right)\right]} \\ &\qquad\qquad \times \pi_{1,2}(e' \mid e_k)\, \mathrm{d}X_{I,1}(\mathbf{s}_I)\Big\}, \end{aligned} \tag{18}$$

$$\begin{aligned} x'_{II,3}(\mathbf{s}'_{II}) \quad &= \quad \frac{1-\xi}{1+\nu}\Big\{\int_{A^2\times E} \mathbf{1}_{\left[a' = a'_{I,2}(\mathbf{s}_I) + a'_{I,4}(\mathbf{s}_I)/n - \tau_E\left(a'_{I,4}(\mathbf{s}_I)/n\right)\right]} \mathrm{d}X_{I,2}(\mathbf{s}_I) \\ &\quad + \int_{A\times E} 1_{\left[a' = a'_{II,2}(\mathbf{s}_{II})\right]} \mathrm{d}X_{II,2}(\mathbf{s}_{II})\Big\}, \end{aligned} \tag{19}$$

[16] For Type I households, since the number of child households per parent household is fixed to n, we don't need the measure $x_{I,i}(\mathbf{s}_I)$ for $i = \{3,4\}$.

and

$$x_{II,4}\left(\mathbf{s}'_{II}\right) = \frac{1}{1+\nu}\int_A \mathbf{1}_{\left[a'=a'_{II,3}(\mathbf{s}_{II})\right]}\,\mathrm{d}X_{II,3}(\mathbf{s}_{II}).$$

The steady-state condition is

$$\begin{aligned} x'_{I,i}(\mathbf{s}_I) &= x_{I,i}(\mathbf{s}_I) \quad \text{for } i \in \{1,2\}, \\ x'_{II,i}(\mathbf{s}_{II}) &= x_{II,i}(\mathbf{s}_{II}) \quad \text{for } i \in \{2,3,4\}, \end{aligned} \tag{20}$$

for all $\mathbf{s}_I \in A^2 \times E$ and $\mathbf{s}_{II} \in A \times E$.

3.5 The Firm's Problem

There is only one perfectly competitive firm in this economy. In a closed economy, the stock of fixed capital K is equal to the sum of total private wealth and the government net wealth W_g. Total labor demand L is equal to total labor supply of households in efficiency units.

$$K = \sum_{i=1}^{2}\int_{A^2\times E}(a_p/n + a_k)\,\mathrm{d}X_{I,i}(\mathbf{s}_I) + \sum_{i=2}^{4}\int_{A\times E} a\,\mathrm{d}X_{II,i}(\mathbf{s}_{II}) + W_g, \tag{21}$$

$$\begin{aligned} L = & \sum_{i=1}^{2}\int_{A^2\times E}\left(e_p\, h_{I,i+2}(\mathbf{s}_I)/n + e_k\, h_{I,i}(\mathbf{s}_I)\right)\,\mathrm{d}X_{I,i}(\mathbf{s}_I) \\ & + \int_{A\times E} e\, h_{II,2}(\mathbf{s}_{II})\,\mathrm{d}X_{II,2}(\mathbf{s}_{II}) + \sum_{i=3}^{4}\int_A e\, h_{II,i}(\mathbf{s}_{II})\,\mathrm{d}X_{II,i}(\mathbf{s}_{II}). \end{aligned} \tag{22}$$

In a closed economy, the gross national product Y is determined by a production function,

$$Y = F(K, AL).$$

The profit maximizing condition of the firm is

$$r + \delta = F_K(K, AL), \qquad w\,(1 + \tau'_S) = F_L(K, AL), \tag{23}$$

where δ is the depreciation rate of capital and τ'_S is the marginal payroll (Social Security and Medicare) tax rate.

In a small open economy, the gross national product Y_S is defined as

$$Y_S = r\,K + w\,L,$$

where K is the sum of total private wealth and the government net wealth, and r and w are international factor prices.

3.6 The Government's Policy Rule

Government tax revenue consists of federal income tax T_F, payroll tax T_S, and federal and state estate taxes T_E. These revenues are calculated as follows:

$$\begin{aligned} T_F &= \sum_{i=1}^{2} \int_{A^2 \times E} \{\tau_F\,(r\,a_p)\,/n + \tau_F\,(w\,e_k\,h_{I,i}(\mathbf{s}_I) + r\,a_k)\}\;\mathrm{d}X_{I,i}(\mathbf{s}_I) \\ &\quad + \int_{A \times E} \tau_F\,(w\,e\,h_{II,2}(\mathbf{s}_{II}) + r\,a)\;\mathrm{d}X_{II,2}(\mathbf{s}_{II}) \\ &\quad + \sum_{i=3}^{4} \int_{A} \tau_F\,(r\,a)\;\mathrm{d}X_{II,i}(\mathbf{s}_{II}), \end{aligned} \tag{24}$$

$$\begin{aligned} T_S &= \sum_{i=1}^{2} \int_{A^2 \times E} \tau_S\,(w\,e_k\,h_{I,i}(\mathbf{s}_I))\;\mathrm{d}X_{I,i}(\mathbf{s}_I) \\ &\quad + \int_{A \times E} \tau_S\,(w\,e\,h_{II,2}(\mathbf{s}_{II}))\;\mathrm{d}X_{II,2}(\mathbf{s}_{II}), \end{aligned} \tag{25}$$

$$\begin{aligned} T'_E &= (1-\lambda) \int_{A^2 \times E} \tau_E\left(a'_{I,3}\,(\mathbf{s}_I)\,/n\right)\;\mathrm{d}X_{I,1}(\mathbf{s}_I) \\ &\quad + \int_{A^2 \times E} \tau_E\left(a'_{I,4}\,(\mathbf{s}_I)\,/n\right)\;\mathrm{d}X_{I,2}(\mathbf{s}_I) \\ &\quad + (1-\lambda) \int_{A} a'_{II,3}\,(\mathbf{s}_{II})\;\mathrm{d}X_{II,3}(\mathbf{s}_{II}). \end{aligned} \tag{26}$$

For simplicity, the wealth left by Type II households is assumed to be included in estate tax. Total tax revenue is the sum of these three tax revenues and Social Security tax from employers, i.e.,

$$T = T_F + 2\,T_S + T_E.$$

The law of motion of the government wealth (debt if it is negative) is

$$W'_g = \frac{1}{1+\mu+\nu}\left\{(1+r)\,W_g + T - C_g - tr_{SS}\,N_{OLD}\right\}, \tag{27}$$

where C_g is government consumption and N_{OLD} is the population of households of age 3 or 4, i.e.,

$$N_{OLD} = \frac{1}{n}\left(1 + \frac{\lambda}{1+\nu}\right).$$

3.7 Recursive Competitive Equilibrium

The definition of a steady-state recursive competitive equilibrium (which is also a Markov perfect equilibrium) of this model is as follows:

Definition 1 ***Steady-State Recursive Competitive Equilibrium:*** *Let s_I and s_{II} be the state of a Type I dynasty and that of a Type II dynasty, respectively, where*

$$\mathrm{s}_I = (a_p, a_k, e_k), \quad \mathrm{s}_{II} = (a, e) \text{ or } (a, -) .$$

Given the time invariant government policy rules,

$$\mathbf{\Psi} = \{\tau_F(.), \tau_{\mathbf{S}}(.), \tau_E(.), tr_{SS}, C_g, W_g\} ;$$

factor prices, r and w; the value functions of households,

$$\{v_{I,i}(\mathrm{s}_I)\}_{i=1}^4 \text{ and } \{v_{II,i}(\mathrm{s}_{II})\}_{i=2}^4 ;$$

the decision rules of households,

$$\left\{c_{I,i}(\mathrm{s}_I), h_{I,i}(\mathrm{s}_I), g_{I,i}(\mathrm{s}_I), a'_{I,i}(\mathrm{s}_I)\right\}_{i=1}^4 \text{ and } \left\{c_{II,i}(\mathrm{s}_{II}), h_{II,i}(\mathrm{s}_{II}), a'_{II,i}(\mathrm{s}_{II})\right\}_{i=2}^4 ;$$

and the measures of dynasties,

$$\{x_{I,i}(\mathrm{s}_I)\}_{i=1}^2 \text{ and } \{x_{II,i}(\mathrm{s}_{II})\}_{i=2}^4$$

are in a steady-state recursive competitive equilibrium if, in every period,

1. a household solves the utility maximization problem, (3) – (11), taking its counterpart's (either its parent's or child's) decision as given,
2. the firm solves the profit maximization problem, and the capital and labor markets clear, i.e., (21) – (23) hold,
3. the government policy rules satisfy (24) – (27),
4. the goods market clears, and
5. the measures of dynasties are constant, i.e., (20) holds.

4 Calibration

The four main parameters — the degree of time preference β and those of parental altruism and child's altruism, η, η_0, and ρ — are determined simultaneously so that the steady-state equilibrium of the model replicates the U.S. economy in terms of four key statistics: the capital-output ratio and the relative sizes of bequests and two-way inter vivos transfers. The functional forms and other parameters are chosen so as to be consistent with macroeconomic and cross-section data in the United States.

4.1 The Choice of Functions and Parameter Values

The model uses the following Cobb-Douglas utility function with constant relative risk aversion (CRRA),

$$u(c_i, h_i) = \frac{\{c_i^{\alpha}(h_i^{\max} - h_i)^{1-\alpha}\}^{1-\gamma} - 1}{1-\gamma},$$

and the Cobb-Douglas production function,

$$F(K_t, A_t L_t) = K_t^{\theta}\,(A_t L_t)^{1-\theta}\,,$$

where $A_t = e^{\mu t}A$ and $L_t = e^{\nu t}L$. Table 2 summarizes the choice of parameters. The model also uses a progressive federal income tax function and a progressive estate tax function.[17] The working ability in this model corresponds to the hourly wage of each household. Three levels of ability e^1, e^2, and e^3 and their probabilities p^1 and p^2 are chosen so that the earnings distribution is consistent with the U.S. data. The correlation of hourly wages of age 1 and age 2 is assumed to be 0.80, and that of an age 2 parent and an age 1 child is assumed to be 0.40.[18] For the fertility shock, I simply assumed that 90 percent of households have about three children and that 10 percent of households have no children.[19] For more details about the choice of functions and parameter values, see Sections 3.1 to 3.3 in Nishiyama (2000).

Table 2: Parameters

Share Parameter for Consumption	α	0.765
Coefficient of Relative Risk Aversion	γ	2.0
Capital Share of Output	θ	0.32
Depreciation Rate of Capital Stock	δ	0.046*
Long-Term Real Growth Rate	μ	0.011*
Population Growth Rate	ν	0.010*
Survival Rate at the End of Age 3	λ	0.546
Fertility Rate at the Beginning of Age 3	ξ	0.9

* Annual Rate

4.2 Target Variables

The target value of the capital-output ratio is 2.81. The capital stock used here is measured by ‘fixed reproducible tangible wealth’ minus ‘durable goods owned by consumers.’ These data are taken from the Survey of Current Business (1997). For the output data, the nominal gross domestic product is used. So, the average capital-output ratio in 1990-96 is 2.81.

[17] Based on the formula by Gouveia and Strauss (1994).

[18] In this calibration, education spending paid by parent households is not included in inter vivos transfers, since the model does not consider the households of age 29 or younger. The correlation of hourly wage of the parent household and its child households, 0.4, is partly due to the parental investment in the children’s schooling.

[19] This setting makes the population growth rate 1 percent per year.

For the relative sizes of bequests and inter vivos transfers, the model uses the flow data from Gale and Scholz (1994) based on the 1986 Survey of Consumer Finance (SCF) in which each head of household was asked if he or she contributed \$3,000 or more to other households during 1983-85. Table 3 shows the annual flows of intergenerational transfers and their relative sizes as a percentage of net wealth.[20]

Table 3: The Annual Flows of Intergenerational Transfers — Gale and Scholz (1994)

Transfer Category	Annual Flow: In Billions of Dollars	Annual Flow: As a Percentage of Net Wealth*
Support Given to:		
Children or Grandchildren	37.74	0.32
Parents or Grandparents	3.44	0.03
Trusts	14.17	0.12
Life Insurance	7.84	0.07
Bequests	105.00	0.88
College Payments	35.29	0.29

*Aggregate net wealth in the 1986 Survey of Consumer Finance was \$11,976 billion.

Table 4 shows the target values used in this calibration. First, both trusts and life insurance are included in bequests from parents to children. The relative size of bequests becomes 1.06 percent of total private wealth. Second, the gifts to grandchildren are included in the gifts to children, and the gifts to grandparents are included in the gifts to parents. Then, the relative size of the inter vivos transfers from parents to children becomes 0.32 percent and that from children to parents becomes 0.03 percent.

Table 4: Target Variables and Values on Bequests

Transfer Category	As a Percentage of Net Wealth	Adjusted* ($\times$17/20.8)
The Annual Flow of Bequests, Trusts, Life Insurance, and a Part of Gifts	1.06	0.87
Inter Vivos Gifts to Children	0.32	0.26
Inter Vivos Gifts to Parents	0.03	0.02

*Adjusted to remove the effects of the supplemental high-income subsample of the SCF.

According to the estimate by Gale and Scholz, the stock of inter vivos transfers as a percentage of net wealth declines from 20.8 percent to 17 percent if they do not include the supplemental high-income subsample of the SCF. To avoid the influence of very wealthy

[20] Excerpted from Table 4, p. 152, Gale and Scholz (1994), and rearranged.

households, the same rate of reduction is simply applied to the flow data. The second column of Table 4 shows the target variables used in this calibration.

4.3 Obtained Main Parameters

Table 5 shows the main parameters obtained through the calibration. For the altruistic parameters η, η_0, and ρ, the first column shows the parameters per recipient and the second column shows the parameters per donor.

Table 5: The Obtained Main Parameters

	Per Recipient		Per Donor	
Annual Time Preference	β	0.934	β	0.934
Parental Altruism (for Bequests)	η	0.529	$\eta\,n$	0.792
Parental Altruism (for Gifts)	η_0	0.323	$\eta_0 n$	0.484
Child's Altruism	ρ	0.025	ρ/n	0.017
Preference on the Next Generation	$\beta^{30}\eta$	0.069	$\beta^{30}\eta n$	0.103

Note: γ (the coefficient of relative risk aversion) = 2.0.

The annual time preference parameter β turned out to be 0.934. In other words, the annual discount rate of a household's own future utility is 6.6 percent.

The degree of parental altruism on the child's well-being after the death of the parent household η is 0.529. This number shows the relative importance of the future utility of each of the child households to the future utility of the parent's own household and determines the level of altruistic bequests. Since a parent household is assumed to have 1.5 child households in this calibration, the degree of altruistic bequest motive is 0.792 in total. This means that a parent household cares about its child households 21 percent less than it cares about itself.

The degree of parental altruism on the child's well-being while the parent household is alive η_0 is 0.323. This number determines the level of inter vivos transfers from a parent household to each of its child households. Considering the number of child households per parent household, the degree of inter vivos transfer motive is 0.484 in total.

The degree of child's altruism on the parent's well-being ρ is 0.025. This calibration considers only tangible gifts and bequests. Since the annual flow of inter vivos transfers from child households to their parent household is only 0.02 percent of total private wealth, the degree of child's altruism turned out to be a very small number. That number would be higher if we also considered the transfer from child households to their parents in the form of services.

In the main calibration, the coefficient of relative risk aversion γ is assumed to be 2.0. Table 6 shows the results under different assumptions of γ from 1.0 to 4.0. If γ were higher (lower), both the parameter of time preference and the degree of altruism would be lower (higher) to keep the capital-output ratio and the relative sizes of bequests and inter vivos transfers at the same level.

Table 6: Obtained Parameters Under Different Assumptions of γ

		Coefficient of Relative Risk Aversion γ		
		1.0	2.0	4.0
Annual Time Preference	β	0.945	0.934	0.906
Parental Altruism (for Bequests)	η	0.707	0.529	0.341
Parental Altruism (for Gifts)	η_0	0.522	0.323	0.127
Child's Altruism	ρ	0.117	0.025	0.001

4.4 The Comparison with Other Dynamic Models

Table 7 compares the obtained parameters with other conventional dynamic models. The first column shows the result of the model developed in this paper (the extended life-cycle model); the second, third, and fourth columns show the results of an infinite horizon model with liquidity constraints, an overlapping generations (OLG) model with accidental bequests, and a pure life-cycle model, respectively.

Table 7: The Comparison with Other Dynamic Models — Obtained Parameters

	Extended Life-Cycle Model	Infinite Horizon Model w/ Liquidity Constraints	OLG Model w/ Accidental Bequests	Pure Life-Cycle Model
Annual Time Preference (β)	0.934	0.942	0.939	0.946
Parental Altruism (η)	0.529	1.000	0.000	0.000
Parental Altruism (η_0)	0.323	1.000	0.000	0.000
Child's Altruism (ρ)	0.025	1.000	0.000	0.000
Capital-Output Ratio	2.81	2.81	2.81	2.81
Bequests*	0.87%	2.75%	0.64%	0.00%
Parental Gifts*	0.26%	1.05%	0.00%	0.00%
Child's Gifts*	0.02%	4.91%	0.00%	0.00%

* Annual Flow as a Percentage of Net Wealth

The model in this paper becomes an infinite horizon model with liquidity constraints when the parameters of altruism are changed to be unity. It becomes an OLG model with accidental bequests when those parameters are changed to zero. Furthermore, it becomes a pure life-cycle model when a perfect annuity market is introduced to the economy and accidental bequests are eliminated.

The infinite horizon model predicts too high a percentage of bequests and inter vivos transfers. The percentage of gifts from children to parents is especially high compared with

Table 8: The Comparison with Other Dynamic Models — Inequality

	Extended Life-Cycle Model	Infinite Horizon Model w/ Liquidity Constraints	OLG Model w/ Accidental Bequests	Pure Life-Cycle Model	U.S. Data(1)
Gini Coefficients					
Earnings	0.610	0.622	0.606	0.605	
Earnings(2)	0.381	0.400	0.375	0.373	(3) 0.51
Income	0.496	0.511	0.496	0.495	
Wealth	0.701	0.782	0.687	0.680	(4) 0.78
Share of Wealth (%)					
Top 1%	14.6	17.0	13.3	12.9	29.6
Top 5%	30.1	38.0	28.6	28.1	53.5
Top 10%	46.5	57.1	45.0	44.3	66.1
Top 20%	71.0	82.0	69.5	67.8	79.5
Top 40%	93.2	97.7	92.2	92.4	92.9
Top 60%	99.0	100.0	98.6	98.7	98.6
Top 80%	100.0	100.0	100.0	100.0	100.4

(1) Sources: Díaz-Giménez et al. (1997), Table 6; Quadrini et al. (1997), Tables 1 & 2.

(2) Households of age 1 and 2.

(3) For household heads ages 35-50.

(4) For married couples only, the number is 0.71 according to the PSID 1989 wealth data.

the U.S. economy.[21] The overlapping generations model with accidental bequests replicates the level of bequests fairly well. Of course, it cannot explain any inter vivos transfers.[22]

To show the performance of the extended life-cycle model, I also compare the distributions of earnings, income, and wealth produced by these four models in Table 8.

In the calibration, the distribution of the working ability of a household is similar to that of the hourly wage of a married couple. The discrepancy between the earnings Gini coefficient of the model and the U.S. data is partly because the U.S. data include the single households. The wealth Gini coefficient generated by the extended model is 0.701, which is lower than that of U.S. data (0.78) but almost the same as that of married couples in the

[21] However, the 4.91 percent predicted by the infinite horizon model may not be too high if we consider services such as informal eldercare.

[22] The results under different assumptions of γ are as follows:

	Infinite Horizon γ		OLG γ		Pure Life-Cycle γ	
	1.0	4.0	1.0	4.0	1.0	4.0
β	0.950	0.920	0.950	0.912	0.953	0.920
Bequests (%)	3.07	2.59	0.50	0.69	0.00	0.00
Parental Gifts (%)	1.11	1.02	0.00	0.00	0.00	0.00
Child's Gifts (%)	5.34	4.69	0.00	0.00	0.00	0.00

Panel Study of Income Dynamics (PSID) data (0.71).

If we look at the wealth distribution, the infinite horizon model seems to best explain the inequality. The top 1 percent and 5 percent shares of wealth are, however, still much lower than those of the U.S. data. But, the extended model explains the wealth shares of the top 40 percent, 60 percent, and 80 percent of households better than the infinite horizon model. Also, it produces better wealth distribution compared with the two overlapping generations models (the OLG model with accidental bequests and the pure life-cycle model).

The results also imply that both the bequest motive and the inter vivos transfer motive tend to intensify inequality. For example, if we compare the infinite horizon model with the pure life-cycle model, the wealth Gini coefficient in the former is higher by about 0.1 than in the latter. But those intergenerational transfers and life-cycle savings cannot fully explain the skewness of the wealth distribution in the United States if we look at the top 1 percent share of the household wealth. We need to add some other forms of assets, such as chunky assets as well as risky assets.

5 Policy Experiments

In this section, first, a 100 percent estate and gift tax is introduced to the economy to show the effects of altruistic bequests and gifts on wealth accumulation and inequality. Next, a perfect annuity market is introduced to show the role of accidental bequests due to lifetime uncertainty. Finally, both a 100 percent estate and gift tax and a perfect annuity market are introduced to show the total effect of intergenerational transfers.

5.1 A 100 Percent Estate and Gift Tax

How much would national wealth be reduced if altruistic bequests and gifts were eliminated? How would the distribution of income and wealth change? In this experiment, the tax rate was raised to 100 percent on both bequests and gifts. Government expenditure and wealth level were assumed to be unchanged, and the increase in tax revenue (from a 100 percent tax on accidental bequests) was transferred to all households in a lump-sum manner. The results are shown in Table 9.

If there were no altruistic bequests and gifts, national wealth would be reduced by 11.1 percent in a closed economy and 16.4 percent in a small open economy.[23] In a closed economy, the interest rate would rise by 1 percentage point.

Earnings inequality would be slightly lower because the estate tax would discourage high-income households from working. Income and wealth inequalities would not change in a closed economy but would be higher in a small open economy. This happens because of the lump-sum transfers. Low-income households would save less as the lump-sum transfer increases. The share of the wealth held by the top 1 percent of households would fall from 14.6 percent to 13.5 percent in a closed economy and to 14.2 percent in a small open economy. That result implies that altruistic bequests and transfers account only slightly for the skewness of wealth distribution in the United States.

[23] In a small open economy, the interest rate and the wage rate are fixed at the levels of the baseline economy.

Table 9: When a 100 Percent Estate and Gift Tax Is Added to the Economy

	Baseline Economy	Closed Economy	Small Open Economy
%Δ National Wealth		-11.1	-16.4
%Δ Labor		0.2	0.5
%Δ GNP		-3.6	-4.9
Δ Interest Rate		1.0	no change
%Δ Wage Rate		-3.7	no change
Gini Coefficients			
Earnings	0.610	0.605	0.605
Earnings (Ages 1 & 2)	0.381	0.373	0.374
Income	0.496	0.498	0.510
Wealth	0.701	0.700	0.712
Share of Wealth (%)			
Top 1%	14.6	13.5	14.2
Top 5%	30.1	29.4	30.5
Top 10%	46.5	46.1	47.8
Top 20%	71.0	71.0	72.8
Top 40%	93.2	93.5	94.2
Top 60%	99.0	98.9	99.0
Top 80%	100.0	100.0	100.0

5.2 A Perfect Annuity Market

If accidental bequests — instead of altruistic transfers — were eliminated from the economy, how much would national wealth be reduced? And, how would the income and wealth inequalities change? In this experiment, a perfect annuity market is introduced to the economy to eliminate precautionary savings for the uncertain life span. Table 10 shows the results.

National wealth would be reduced by a modest 0.2 percent in a closed economy and 1.0 percent in a small open economy. The change in the interest rate is negligible. The effect of a perfect annuity market on earnings and income inequalities is very small. But, wealth inequality would increase significantly. The Gini coefficient would rise about 0.02 points. The share of wealth held by the top 1 percent of households would rise from 14.6 percent to 18.7 percent in a closed economy and to 18.8 percent in a small open economy.

The introduction of a perfect annuity market would reduce precautionary savings for uncertain life span. But, at the same time, it would increase the marginal value of wealth at the beginning of age 3 when the life-cycle saving is highest for most households. This result implies that accidental bequests (and lifetime uncertainty) do not explain the skewness of wealth distribution in the United States.

Even though the steady-state wealth level and GNP are lower than those of the baseline economy, the introduction of a perfect annuity market would improve welfare. Table 10 also shows the welfare changes based on the compensating variation wealth measure and the

Table 10: When a Perfect Annuity Market Is Added to the Economy

	Baseline Economy	Closed Economy	Small Open Economy
%Δ National Wealth		-0.2	-1.0
%Δ Labor		0.4	0.4
%Δ GNP		-0.2	0.0
Δ Interest Rate		0.1	no change
%Δ Wage Rate		-0.2	no change
Welfare Changes (%)			
Compensating Variation$^{(1)}$		3.9	2.6
Equivalent Variation$^{(2)}$		4.0	2.7
Gini Coefficients			
Earnings	0.610	0.608	0.608
Earnings (Ages 1 & 2)	0.381	0.379	0.379
Income	0.496	0.499	0.500
Wealth	0.701	0.720	0.721
Share of Wealth (%)			
Top 1%	14.6	18.7	18.8
Top 5%	30.1	33.7	33.8
Top 10%	46.5	49.4	49.5
Top 20%	71.0	73.0	73.1
Top 40%	93.2	94.2	94.2
Top 60%	99.0	99.2	99.2
Top 80%	100.0	100.0	100.0

(1) (Total Wealth / Compensating Wealth – 1)$\times$100

(2) (Equivalent Wealth / Baseline Total Wealth – 1)$\times$100

equivalent variation measure. Households would be on average better off by 3.9–4.0 percent in a closed economy and 2.6–2.7 percent in a small open economy. For the computation of these welfare measures, see Appendix B.

The effect of a perfect annuity market would be much larger if households were not altruistic or, equivalently, if a 100 percent estate tax were introduced. In the economy without any altruistic transfers, the introduction of a perfect annuity market would reduce national wealth by 3.4 percent in a closed economy and by 4.6 percent in a small open economy. In the presence of intergenerational transfers, both altruistic bequests and inter vivos transfers would be a buffer for the risk of uncertain life span. If these altruistic transfers were prohibited for some reason, the risk of lifetime uncertainty would be larger, so the effect of perfect annuity markets would be larger.

5.3 A Perfect Annuity Market with a 100 Percent Estate and Gift Tax

In total, how much would national wealth be reduced if there were no intergenerational transfers? How much would they affect income and wealth inequalities? Table 11 shows the effect of both a perfect annuity market and a 100 percent estate and gift tax.

Table 11: When a 100 Percent Estate and Gift Tax and a Perfect Annuity Market Are Added to the Economy

	Baseline Economy	Closed Economy	Small Open Economy
%Δ National Wealth		-14.1	-20.3
%Δ Labor		0.3	0.7
%Δ GNP		-4.6	-6.0
Δ Interest Rate		1.3	no change
%Δ Wage Rate		-4.8	no change
Gini Coefficients			
Earnings	0.610	0.603	0.604
Earnings (Ages 1 & 2)	0.381	0.371	0.372
Income	0.496	0.491	0.507
Wealth	0.701	0.687	0.704
Share of Wealth (%)			
Top 1%	14.6	13.1	13.8
Top 5%	30.1	28.8	30.4
Top 10%	46.5	45.3	47.5
Top 20%	71.0	68.9	71.4
Top 40%	93.2	92.7	93.6
Top 60%	99.0	98.7	98.9
Top 80%	100.0	100.0	100.0

National wealth would be reduced by 14.1 percent in a closed economy and by 20.3 percent in a small open economy. The interest rate would rise 1.3 percentage points in a closed economy. Earnings inequality would be slightly lower, but income and wealth Gini coefficients would fall in a closed economy and rise in a small open economy. The share of wealth of the top 1 percent of households would fall from 14.6 percent to 13.1 percent in a closed economy and to 13.8 percent in a small open economy. This result implies that intergenerational transfers account to some extent for the skewness of wealth distribution in the United States.

6 Concluding Remarks

Through the calibration of the model to the U.S. economy, I obtained the degrees of intergenerational altruism. The parent's motive for inter vivos transfers η_0 turned out to be smaller than its altruistic bequest motive η. There are several possible explanations for this result.

First, a parent household may discount its child household's future utility less than the child discounts its own future utility. In that case, even though the child household cares about its current consumption relatively strongly, the parent household does not want to make a gift to allow the child to consume as much as it wants. Second, in the real economy, the working ability and the effort level of a child household may not be fully observable by its parent. In that case, the parent household would transfer less in the form of inter vivos transfers in order to avoid the moral hazard problem caused by intergenerational risk sharing. Third, this result seems to support the altruistic model of transfers rather than "joy-of-giving" type models because if the gift motive were selfish, parents would make more gifts while they were alive.

Based on the transfer data in the Survey of Consumer Finance, the child's inter vivos transfer motive ρ turned out to be very small. But, there are other unmeasured gifts by children that should be considered, such as informal caregiving. According to Arno, Levine, and Memmott (1999), the national economic value of informal caregiving was estimated at \$196 billion in 1997. If we consider a part of that caregiving as the gifts from child households to their parent households, the degree of child's altruism will be much higher than the result obtained in the model.[24]

The introduction of inter vivos transfers and two-way altruism does not change the effect of intergenerational transfers on wealth accumulation very much. The results are similar to those in the previous paper (bequests only). National wealth would be reduced by 14 percent in a closed economy and by 20 percent in a small open economy if there were no intergenerational transfers.

Regarding the wealth inequality, the effect of these transfers is not very strong. Based on the parameters chosen and obtained in this paper, eliminating transfers would lower the wealth Gini coefficient in a closed economy, but increase it slightly in a small open economy. When we look at the share of wealth held by the top 1 percent of households, a 100 percent estate and gift tax (to eliminate altruistic transfers) would reduce that share, but a perfect annuity market (to eliminate accidental bequests) would rather increase inequality. In other words, bequests and inter vivos transfers seem to contribute to the skewness of the wealth distribution in the United States, but are not likely to be the main cause of it.

Appendixes

A The Computation of Equilibria

The equilibria of the model are obtained numerically. The state space of a dynasty is discretized. To find the optimal end-of-period wealth, the model uses the Euler equation method and bilinear (for Type I households) or linear (for Type II households) interpolation of marginal value functions in the next period.

In this appendix, I only explain how to find the decision rule of households. For other detatils of the computation, see the appendix to Nishiyama (2000).

[24]Annual flow of \$196 billion in 1997 corresponds to 0.86 percent of the (estimated) total private wealth. If we consider 70 percent of this amount as inter vivos gifts from child households to their parent households, the degree of child's altruism, ρ, becomes 0.158.

A.1 The Decision Rule of Households

The algorithm to find the decision rule of Type I households is as follows. For simplicity, the explanation is abstracted from population growth, productivity growth, and lifetime uncertainty.

1. Set the initial numbers of marginal values $\{\mathbf{v}'^0_{I,i}(\widehat{\mathbf{s}}_I)\}_{i=2}^4$.

2. For each $(\widehat{\mathbf{s}}_I, i) \in \widehat{A}^2 \times \widehat{E} \times \{1,2,3,4\}$ find the decision rule of all households, $\mathbf{d}_{I,i}(\widehat{\mathbf{s}}_I) = \mathbf{d}_p$ or $\mathbf{d}_k$, taking government policy rules $\mathbf{\Psi} = \{\tau_F(.), \tau_S(.), \tau_E(.), tr_{SS}, C_g, W_g\}$, factor prices (r^0, w^0), and the marginal values as given.

   (a) Set the initial values on the decision of the child household $\mathbf{d}_k^0 = (c_k^0, h_k^0, g_k^0, a_k'^0)$.

   (b) Given the decision of the child household $\mathbf{d}_k^0$, find the optimal decision of its parent household $\mathbf{d}_p^0 = (c_p^0, h_p^0, g_p^0, a_p'^0)$.

      i. Set the initial value of the parent's end-of-period wealth $a_p'^0(\mathbf{d}_k^0)$ and the gift to its child household $g_p^0(a_p'^0, \mathbf{d}_k^0)$.

      ii. Find the level of consumption and working hours, $c_p^0(g_p^0, a_p'^0, \mathbf{d}_k^0)$ and $h_p^0(g_p^0, a_p'^0, \mathbf{d}_k^0)$, using the marginal rate of substitution of c_p^0 for h_p^0 and after-tax marginal wage rate.

      iii. Compare the marginal utility of consumption of its own and of its child household. If $u_1(c_p^0, h_p^0) \geq \eta_0\, u_1(c_k^0, h_k^0)$ with equality holds when $g_p^0 > 0$, go to Step (iv). Otherwise, replace g_p^0 with g_p^1 that solves $\varepsilon = \arg\min |\eta_0\, u\left(c_k^0 + \varepsilon, h_k^0\right) - u\left(c_p^0 - \varepsilon, h_p^0\right)|$ subject to $g_p^1 = g_p^0 + \varepsilon \geq 0$, and return to Step (ii).

      iv. Check the Euler equation of the parent household. If
      $$\frac{\partial}{\partial c_p} u(c_p^0, h_p^0) \geq \begin{cases} \beta E \frac{\partial}{\partial a_p'} v_{I,i+1}(\widehat{\mathbf{s}}_I') & \text{(if } i=3\text{)} \\ \beta E \eta \frac{\partial}{\partial a_p'} v_{I,i-2}(\widehat{\mathbf{s}}_I') & \text{(if } i=4\text{)} \end{cases}$$
      with equality holds when $a_p'^0 > 0$, go to step (c). Otherwise, replace $a_p'^0$ with $a_p'^1$ where
      $$a_p'^1 = \begin{cases} \arg\min \left|\beta E \frac{\partial}{\partial a_p'} v_{I,i+1}(\widehat{\mathbf{s}}_I') - \frac{\partial}{\partial c_p} u(c_p^0, h_p^0)\right| & \text{(if } i=3\text{)} \\ \arg\min \left|\beta E \eta \frac{\partial}{\partial a_p'} v_{I,i-2}(\widehat{\mathbf{s}}_I') - \frac{\partial}{\partial c_p} u(c_p^0, h_p^0)\right| & \text{(if } i=4\text{)} \end{cases}$$
      subject to $a_p'^1 \geq 0$, and return to step (ii).

   (c) Similarly, given the decision of the parent household $\mathbf{d}_p^0$ obtained in step (b), find the optimal decision of its child household $\mathbf{d}_k^1 = (c_k^1, h_k^1, g_k^1, a_k'^1)$.

   (d) Compare the new decision of the child household, $\mathbf{d}_k^1$, with the old one, $\mathbf{d}_k^0$. If the difference is sufficiently small, then go to step (e). Otherwise, replace $\mathbf{d}_k^0$ with $\mathbf{d}_k^1$ and return to step (b).

   (e) Compute the marginal values $(\mathbf{v}'^1_{I,4}(\widehat{\mathbf{s}}_I), \mathbf{v}'^1_{I,2}(\widehat{\mathbf{s}}_I))$ or $\mathbf{v}'^1_{I,3}(\widehat{\mathbf{s}}_I)$ using $(\mathbf{d}_p^0, \mathbf{d}_k^0)$.

3. Compare the new marginal values $\{\mathbf{v}'^1_{I,i}(\widehat{\mathbf{s}}_I)\}_{i=2}^4$ with $\{\mathbf{v}'^0_{I,i}(\widehat{\mathbf{s}}_I)\}_{i=2}^4$. If the difference is sufficiently small, then stop. Otherwise, replace $\{\mathbf{v}'^0_{I,i}(\widehat{\mathbf{s}}_I)\}_{i=2}^4$ with $\{\mathbf{v}'^1_{I,i}(\widehat{\mathbf{s}}_I)\}_{i=2}^4$ and return to step 2.

B The Computation of Welfare Measures

Since the labor supply of households is endogenous in this paper, I constructed the following welfare measures based on the wealth level of households and the government. The results are shown in Section 5.2.

B.1 The Compensating Variation

Let $v^0_{I,i+2}(a_p, a_k, e_k)$, $v^0_{I,i}(a_p, a_k, e_k)$, and $v^0_{II,j}(a, e)$ be the value functions of a Type I parent household, a Type I child household, and a Type II household, respectively, in the baseline economy, where $i \in \{1, 2\}$ and $j \in \{2, 3, 4\}$. Let $X^0_{I,i}(a_p, a_k, e_k)$ and $X^0_{II,j}(a, e)$ be the cumulative measures of Type I households and Type II households, respectively, in the baseline economy. Let $v^1_{I,i+2}(a_p, a_k, e_k)$, $v^1_{I,i}(a_p, a_k, e_k)$, and $v^1_{II,j}(a, e)$ be the corresponding value functions in the alternative economy. Suppose that the functions $a^C_{I,i+2}(a_p, a_k, e_k)$, $a^C_{I,i}(a_p, a_k, e_k)$, and $a^C_{II,j}(a, e)$ solve

$$v^1_{I,i+2}\left(a^C_{I,i+2}, a^C_{I,i}, e_k\right) = v^0_{I,i+2}(a_p, a_k, e_k),$$

$$v^1_{I,i}\left(a^C_{I,i+2}, a^C_{I,i}, e_k\right) = v^0_{I,i}(a_p, a_k, e_k),$$

$$v^1_{II,j}\left(a^C_{II,j}, e\right) = v^0_{II,j}(a, e)$$

for all $(a_p, a_k, e_k) \in A^2 \times E$ and $(a, e) \in A \times E$. Then the compensating wealth K^C is defined as

$$K^C = \sum_{i=1}^{2} \int_{A^2 \times E} (a^C_{I,i+2}/n + a^C_{I,i})\, \mathrm{d}X^0_{I,i}(\mathbf{s}_I) + \sum_{j=2}^{4} \int_{A \times E} a^C_{II,j}\, \mathrm{d}X^0_{II,j}(\mathbf{s}_{II}) + W^0_g,$$

where W^0_g is the government wealth in the baseline economy. Let K^1 be the total wealth (including the government wealth) in the alternative economy. The welfare change (in percent) measured by the compensating variation is defined as $\left(K^1/K^C - 1\right) \times 100$. The alternative economy is potentially Pareto preferred to the baseline economy if $K^1 > K^C$.

B.2 The Equivalent Variation

Let $X^1_{I,i}(a_p, a_k, e_k)$ and $X^1_{II,j}(a, e)$ be the cumulative measures of Type I households and Type II households, respectively, in the alternative economy. Suppose that the functions $a^E_{I,i+2}(a_p, a_k, e_k)$, $a^E_{I,i}(a_p, a_k, e_k)$, and $a^E_{II,j}(a, e)$ solve

$$v^0_{I,i+2}\left(a^E_{I,i+2}, a^E_{I,i}, e_k\right) = v^1_{I,i+2}(a_p, a_k, e_k),$$

$$v^0_{I,i}\left(a^E_{I,i+2}, a^E_{I,i}, e_k\right) = v^1_{I,i}(a_p, a_k, e_k),$$

$$v^0_{II,j}\left(a^E_{II,j}, e\right) = v^1_{II,j}(a, e)$$

for all $(a_p, a_k, e_k) \in A^2 \times E$ and $(a, e) \in A \times E$. Then the equivalent wealth K^E is defined as

$$K^E = \sum_{i=1}^{2} \int_{A^2 \times E} (a^E_{I,i+2}/n + a^E_{I,i}) \, \mathrm{d}X^1_{I,i}(\mathbf{s}_I) + \sum_{j=2}^{4} \int_{A \times E} a^E_{II,j} \, \mathrm{d}X^1_{II,j}(\mathbf{s}_{II}) + W^1_g,$$

where W^1_g is the government wealth in the alternative economy. Let K^0 be the total wealth (including the government wealth) in the baseline economy. The welfare change (in percent) measured by the equivalent variation is defined as $\left(K^E/K^0 - 1\right) \times 100$. The alternative economy is potentially Pareto preferred to the baseline economy if $K^E > K^0$.

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