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Complementary Putty-Clay Capital and Its Implications for Modeling Business Investment and Measuring Income from Intangible Capital

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Abstract

This paper presents two basic innovations: a new type of putty-clay capital, used to model business investment; and the use of investment to identify shares of capital income and market value accruing to tangible and intangible capital. Unlike in existing putty-clay models, new capital is complementary with existing capital, allowing investment to be specified as a function of the productivity of existing capital and of Tobin's q. In contrast to Tobin's q models using neoclassical (putty-putty) capital, q is not a sufficient statistic for investment, which also depends on the growth of aggregate demand. The model implies that investment in the types of capital measured in the national income and product accounts, whether equipment, structure, inventory, or intellectual property product, depends on the market value of existing assets of those types. Thus, investment can be used to identify those assets' shares of market value and capital income. The market value and income of unmeasured intangible capital, which includes organizational capital, reputation, and market power, can be estimated by subtraction. I find a sharp increase in the income of unmeasured intangible capital since 2000. Relatively weak investment in measured capital in recent years can be reconciled with high levels of asset prices and capital income because those high levels are due to unmeasured intangible capital and so do not affect the incentive to invest in measured capital.

1 Introduction

There are two standard ways of modeling capital: as putty-clay or as putty-putty. If capital is putty-clay, the capital-labor ratio of a new unit of capital can be freely chosen (or putty) before the unit is purchased, but is fixed (or clay) afterwards. For example, before a business buys a truck, it can decide on the quality, or dollars, of truck it wants one driver to operate, but after the truck is purchased, quality per driver operating the truck is fixed. In contrast, if capital is putty-putty, the capital-labor ratio of an individual unit of capital remains variable after the unit is purchased. In that model, the firm varies the number of workers operating a unit of capital based on the relative cost of workers and capital.

Each of those approaches has its problems. In the putty-putty model, which is the dominant model in economics because of its mathematical simplicity, the problems are empirical. In its Tobin's q variant, the putty-putty model predicts that q (the ratio of the market value of capital to its replacement cost) should be a sufficient statistic for investment, but empirical work finds cash flow to be at least as important. In Jorgenson's (1963) neoclassical variant, the putty-putty model predicts that investment should respond symmetrically to a 1 percent increase in output and a 1 percent decrease in the cost of capital, but empirical work finds the short-run response of investment to output much greater than the response to the cost of capital. However, in traditional putty-clay models, the problem is in connecting theory with empirics. The model implies that the productivity of new capital cannot be estimated from the productivity of existing capital, so an ad hoc assumption is needed to estimate the model.

The first major innovation in this paper, complementary putty-clay capital (CPC), solves both types of problems that arise in the modeling of putty-clay capital. In the CPC model, the capital input to production is a Cobb-Douglas composite of the quality of each unit of capital, properly scaled. The complementarity between new and existing capital solves the problem of traditional putty-clay models by making the productivity of new capital observable from the productivity of existing capital. The model solves the problem of the putty-putty Tobin's q model by making investment depend on growth of demand (for which cash flow appears to serve as a proxy) as well as on q. It solves the problem of the neoclassical model because investment responds differently in the short run to changes in demand than to changes in the cost of capital, as in traditional putty-clay models. The CPC model in this paper updates the model presented in Lasky (2007) to incorporate Tobin's q.

The second major innovation in this paper is the use of investment to identify the income and market value of the various types of capital. The national income and product accounts (NIPAs) currently measure investment in tangible capital (equipment, software, and inventories) and in some types of intangible capital (intellectual property products). I refer to all

of those as "measured" capital. In addition, the market value of a firm includes the value of capital unmeasured by the NIPAs, known as unmeasured intangible capital (UIC). Such capital can take many forms, such as reputation, organizational capital, and market power. Furthermore, these components can make up a large fraction of a firm's market value. For example, at the end of its 2013 fiscal year, Apple Inc. reported tangible and certain intangible assets of about $30 billion, but the market value of its equity less its financial assets was about $250 billion. Because investment in measured capital depends on the income it is expected to generate, I can determine the portions of capital income accruing to measured capital and to UIC by combining information from estimating investment with data for total market value and capital income. Hall (2000) and McGrattan and Prescott (2010) also measure intangible capital, but using very different methodologies from mine.

The model has important empirical implications. I find that the increase in capital's share of income since about 2000 is due to increased income of UIC rather than to a substitution of capital for labor. That helps explain why business investment has remained weak in recent years despite high levels of capital income and market value: those high levels are due to high levels of UIC and so do not affect the incentive to invest in measured capital.

The remainder of the paper is organized as follows. Section 2 reviews the standard models of investment. Section 3 introduces complementary putty-clay capital. Section 4 derives a basic model of investment and Section 5 compares the properties of that model to those of standard models of investment. Section 6 modifies the basic model to better square with the real world, while Section 7 describes the estimation procedure. Section 8 discusses the empirical results, shows that cash flow is insignificant when added to the model, apportions capital income and market value between measured capital and UIC, and calculates a better measure of q. Section 9 concludes.

2 Standard Models of Investment

The two most commonly used models of investment in the academic literature—Tobin's q and Jorgenson's neoclassical model—are closely related. Most importantly from the standpoint of this paper, both models assume that capital is homogeneous, or putty-putty. Under that assumption, businesses can costlessly alter the capital-labor ratio embodied in existing capital. The model typically used by economic forecasters—the standard putty-clay model—differs from putty-putty models by assuming that businesses cannot alter the capital-labor ratio embodied in existing capital.

2.1 Tobin's q

The q theory of investment, proposed by Keynes (1936) and developed by Tobin (1969), analyzes the investment decision as a function of q. Marginal q is the ratio of the market value of an additional unit of capital to its replacement cost. The higher that ratio, the greater is investment. Although marginal q is unobservable, Hayashi (1982) showed that, under certain conditions, average q (the ratio of the market value of all capital to its replacement cost) equals marginal q. When tax treatment is taken into account, Hayashi shows that

$$\frac{I}{K} = h\left(\frac{q}{1 - itc - uz}\right), \tag{1}$$

where I is investment, K is the capital stock, itc is the investment tax credit, u is the corporate tax rate, z is the present discounted value (PDV) of depreciation allowances for new capital, and $h' > 0$. The expression $q/(1 - itc - z)$ is often called tax-adjusted q. Summers (1981), by starting with a different treatment of debt and equity, finds that investment also depends on the fraction of investment that firms finance with debt.

In most models, q differs from 1 because capital is costly to adjust. In that case, q represents the shadow value of being able to increase capital and dh/dq is inversely related to the cost of adjustment. In Abel and Eberly's (2011) model, however, there are no adjustment costs and marginal q is always 1, but average q is still correlated with investment through expected growth of revenue.

Although most theoretical models find that q should be a sufficient statistic for investment, most empirical work, including Fazzari, Hubbard, and Petersen (1988) and Kaplan and Zingales (1997), finds a larger role for cash flow than for q. Fazzari et al claim that cash flow measures the easing of liquidity constraints that prevent firms from investing as much as they would like, but Kaplan and Zingales dispute that explanation. Another possible justification for cash flow is that it proxies for marginal q; however, beginning with Abel and Blanchard (1986), most researchers attempting to develop better proxies for marginal q have found that variables such as cash flow continue to be empirically significant. Erickson and Whited (2000) is an exception.

Even with the inclusion of cash flow, the q model has performed poorly since the end of the Great Recession. Both q and cash flow were unusually strong in 2010-2013, but the ratio of investment to the capital stock was unusually weak (see Figure 1).

2.2 The Neoclassical Model

The neoclassical model of investment pioneered by Jorgenson (1963) derives investment from the optimal stock of capital. Jorgenson found that the optimal real capital stock K^* is given by

$$K^* = \alpha \frac{Y}{v}, \tag{2}$$

where α is capital's coefficient in production, Y is real output, and v is the real rental cost of capital, which depends on the real price and tax treatment of new capital, its depreciation rate δ, and the rate of interest. Because it is costly to adjust K^*, real gross investment I equals replacement demand δK plus a lag function $\beta(L)$ of past changes in K^*:

$$I_t = \beta(L)\left[K_t^* - K_{t-1}^*\right] + \delta K_t.$$

Thus, investment depends positively on past changes in output and negatively on past changes in the real cost of capital v. Abel (1980) showed that the q model and a neoclassical model with adjustment costs are special cases of the same general putty-putty model, in which the q model is derived by assuming infinitesimal time intervals and the neoclassical model is derived by assuming long time intervals.

Empirically, the neoclassical model is preferable to the q model because it provides a more plausible rationale for cash flow, as a proxy for output, but it still has a major shortcoming. The theory says that the elasticities of investment with respect to output and to the cost of capital should be of equal magnitude. However, researchers generally find that the cost of capital has a much smaller effect on investment in the short run than does output (Eisner and Nadiri (1968), Coen (1971), and Chirinko (1993)), or no effect at all (Eisner (1969)).

2.3 Standard Putty-Clay Models

Putty-clay models answer the question of why the elasticities of investment with respect to output and the cost of capital are not of equal magnitude in the short run. In those models, originally developed by Johansen (1959), Solow (1962), and Phelps (1963), firms can choose the capital-labor ratio embodied in new capital but cannot vary it once the capital is placed in service. Thus, capital is "putty" as the firm decides on the capital-labor ratio and "clay" after the capital has been produced. In putty-clay models, the number of new units of capital depends on growth of output, while the amount of capital in a new unit of capital depends on the cost of capital. Investment depends on the *change* in output but on the *level* of the cost of capital. Consequently, investment responds less in the short run to a change in the

cost of capital than to a change in output. Bischoff (1971) estimated a putty-clay model of investment and found the long-run elasticity of the cost of capital to be -1, consistent with equation (2).

Unfortunately, standard putty-clay models solve the empirical problem of the neoclassical model by opening a gap between the theory and the estimated investment equation. The basic problem is that standard putty-clay models assume that the productivity of new capital is independent of the productivity of existing capital, and so cannot be estimated from aggregate data. With no way to estimate the productivity of new capital, it is impossible to determine how much new capital is required to satisfy a given increase in demand without making ad hoc assumptions. By assuming that new capital is complementary with existing capital, the model in this paper provides a link between the productivity of new capital and the observed productivity of existing capital.

3 Properties of Complementary Putty-Clay Capital

CPC capital differs from other descriptions of capital in the way new investment is added to the existing capital stock. As in standard putty-clay models, firms invest in discrete units of capital—trucks, buildings, computers, and so on. The amount of capital embodied in a new unit of capital is its "quality," denoted k. Unlike in standard putty-clay models, the quality of the aggregate capital stock is specified as a Cobb-Douglas function of the quality of individual units. That means the productivity of new capital is positively related to the quality of existing units of capital, even though there are decreasing returns to the quality of a new unit of capital. For example, a personal computer is more productive the better the software it is paired with, but there is a decreasing return to each additional terabyte of memory. Across businesses, capital is more productive in one business if its suppliers use better capital. In contrast, traditional putty-clay models are specified so that the productivity of new capital is independent of the quality of existing capital. In putty-putty models, such as the neoclassical model, the productivity of new capital is lower the greater is the quality of existing capital.

3.1 Specification of Complementary Putty-Clay Capital

The quality of a unit of capital, k, and the number of workers using it remain constant over its service life. Personal computers, trucks, and chairs are good examples: the amount of capital embodied in each is usually fairly constant over its service life, and only one person uses or operates each one at a given time. For simplicity, the basic CPC model assumes a

single type of capital (e.g., widgets) and assumes that units of capital are normalized so that each worker uses one unit of capital. Those assumptions are relaxed later in the paper.

Aggregate quality per unit of capital $\bar{k}$ is a geometric average of the quality of each unit of capital. Assuming that there are M units (and thus M workers) indexed from 1 to M with unit j having quality k_j, the aggregate quality per unit is given by

$$\log\left(\bar{k}\right) = \frac{1}{M}\sum_{j=1}^{M}\log\left(k_j\right). \tag{3}$$

One can think of equation (3) as a Cobb-Douglas function combining the amount of capital in each unit of capital:

$$\bar{k} = k_1^{1/M} k_2^{1/M} ... k_M^{1/M}.$$

The capital input to production, K, is aggregate quality per unit times the number of units, or $\bar{k}M$.[1]

That method of aggregation accounts for why the productivity of new capital depends positively on the quality of existing capital. The product of an additional increment of quality for the first unit of capital, k_1, is

$$\frac{dY}{dk_1} = \frac{dY}{dK}\frac{K/M}{k_1},$$

where Y denotes output. This function is increasing in K for any CES production function with a positive elasticity of substitution between labor and capital, including the Cobb-Douglas production function. Consequently, the productivity of new capital increases with the quality of existing capital. However, the marginal product of additional quality is decreasing in k_1, so there are decreasing returns to additional investment.

CPC capital differs from other types of putty-clay capital in that new units of capital are complementary with existing capital, making it possible to determine the productivity of new capital from that of existing capital. Johansen (1959), Phelps (1963), Solow et al (1966), Calvo (1976), and Gilchrist and Williams (2005) all assume that the output of a given unit of capital is constant over its service life, and thus that the productivity of new capital is independent of the productivity of existing capital, i.e., dY/dk_1 is independent of K. That makes it impossible to determine the productivity of new capital using aggregate data for output or productivity. Ando et al (1974) assume that the required labor input per unit of

[1] Aruga (2009) presents a model in which the aggregate stocks of two different types of capital are complementary. In that model, however, individual units of one type of capital are only complementary with the other type of capital.

existing capital rises at a constant rate per year, but they cannot determine the productivity of new capital using aggregate data. In all of those models, new capital is neither a complement nor a substitute for existing capital.

CPC capital differs even more from the q and neoclassical models in that new capital is a substitute for existing capital in those models. In those models, dY/dk_1 equals dY/dK, which is decreasing in K. Businesses can reallocate existing capital across workers arbitrarily based on the relative costs of capital and labor. For example, if the cost a given type of personal computer falls from \$1,000 to \$500, a business owning two of them can respond either by puttying the two together into one computer or by buying \$1,000 of new computer, dividing it in two, and puttying each half together with an existing computer.

3.2 Investment and the Capital Stock

The quality of each unit of capital depends on the expectations prevailing when a business first added it to the capital stock, i.e., when the investment takes place. Consequently, aggregate capital per unit depends on the quality of each vintage of existing capital and on the number of units of each vintage. I model investment as done by a representative firm, so each unit of the same vintage has the same quality.

At this point, two somewhat unrealistic simplifying assumptions are required. First, the model uses continuous time. Because of the discrete nature of putty-clay capital, that can only be an approximation. That approximation is necessary, however, to relate investment to Tobin's q. Second, in order to hold the quality of each unit of capital constant over its service life, depreciation takes the form of the retirement of existing units of capital at constant rate δ. Realistically, the productivity of old units of capital can fall so far below that of new units that firms could profitably discard and replace those older units even while they are still functional. Such endogenous replacement would greatly complicate the analysis and so is not considered in this paper. In addition, one might think that the rate of retirement increases as capital ages. I account for that in the empirical section.

If F_i is the flow of new units of capital at time i, then aggregate quality per unit of capital is given by

$$\log\left(\bar{k}_t\right) = \frac{\int_{i=t-\infty}^{t} F_i \exp\left(-\delta\left(t-i\right)\right) \log\left(k_i\right) di}{M_t}, \tag{4}$$

where

$$M_t = \int_{i=t-\infty}^{t} F_i \exp\left(-\delta\left(t-i\right)\right) di.$$

Real investment, I_t, is the product of the flow of new units of capital and their quality:

$$I_t = F_t k_t.$$

The stock of capital at time t (K_t) is aggregate quality per unit of capital times the number of units of capital:

$$K_t = \bar{k}_t M_t. \tag{5}$$

The fact that investment is the product of two components—the number of new units and their quality—is another key difference between CPC capital and putty-putty capital. The first component—the number of new units, F—depends on the *growth* of the factors determining M, the number of units of capital. However, the second component—the quality of new units, k—depends on the *level* of the factors determining it. Thus, for example, a 1 percent increase in output (which increases M by 1 percent) and a 1 percent decrease in the cost of capital (which increases k by roughly 1 percent) have qualitatively different impacts on investment. Putty-putty capital does not differentiate between the number of units of capital and their quality, so investment responds the same to a 1 percent increase in output as to a 1 percent decrease in the cost of capital.

To determine the growth rate of the stock of capital, I differentiate equations (4) and (5) with respect to t yielding:

$$\frac{d\left(K_t\right)}{dt} = K_t \left[\frac{F_t}{M_t} \log\left(\frac{k_t}{\bar{k}_t}\right) + \frac{dM_t/M_t}{dt}\right]. \tag{6}$$

The first portion of the expression in brackets represents capital deepening (increases in the quality of existing units of capital) while the second portion represents capital widening (increases in the number of units of capital). The rate of capital deepening depends on the ratio of the quality of new units of capital relative to the quality of existing units ($k_t/\bar{k}_t$). That ratio has a larger effect on the growth of the capital stock the greater is the flow of new units relative to the existing stock (F_t/M_t).

4 Investment in the Basic Model

I model investment by examining the profit-maximization decisions of a monopolistically competitive representative firm. That firm takes aggregate demand, the average price of output, the price of new capital, tax law, the wage rate, and the market value of capital as given. The firm does not know the future values of those variables with certainty, but rather forms expectations. However, to simplify the math, I assume that a function of the

expected values of one or more variables equals the expected value of the same function of those variables, i.e., if $E(x)$ denotes expectations of a set of variables x and $f(x)$ is a function of x, then $f(E(x)) = E(f(x))$.

The firm maximizes profits across several variables. Regarding the labor input to production, the firm chooses the number of workers and hours worked per week. That decomposition of labor hours is necessary because the response of investment to a change in output per worker depends on the source of that change. An increase in output per worker due to higher total factor productivity (TFP) leads to an increase in the quality of new units of capital, but not in the number of units of capital. However, an increase in output per worker in response to a demand shock that boosts weekly hours signals that the firm will be hiring more workers, and thus boosts the number of new units of capital but not their quality.

In addition, the firm chooses the number of new units of capital and the quality of those units. Together, the choice of labor and capital inputs to production determines the firm's output. Because the firm is monopolistically competitive, the firm's output determines the price it receives and thus its profits.

The basic model assumes a single type of capital and a uniform production function for all firms. Below, I explore investment in the presence of many types of capital, including inventories and land, and production in one sector (mining) that is more capital-intensive than production in the rest of the economy. A uniform production function is conceptually awkward in that the ratio of capital goods prices to the aggregate price level can change over time. For simplicity, I assume that firms produce a single type of intermediate output, which is then sold to a firm that transforms it into various types of final goods and services, including capital. A more realistic model, in which the representative firm produces many types of goods and services using different levels of total factor productivity, would ultimately produce the same result but with a great deal more complexity.

4.1 The Production Function

The production function is a Cobb-Douglas function of capital and labor with constant returns to scale, modified to accommodate both labor inputs, a putty-clay capital stock, a time-varying coefficient of capital, and temporary differences between the number of workers and the number of units of capital. At time t, that production function is

$$Y_t = A_t^* K_t^{\alpha_t} L_t^{1-\alpha_t}, \tag{7}$$

where Y is output, A^* is TFP (A) adjusted for variations in capital's coefficient in the production function α, K is the capital input to production, and L is the labor input.[2]

TFP is adjusted for variations in α so that, at the margin, the firm is indifferent to changing capital's coefficient in production. Except in the rare case that $K = L$, a change in α results in a spurious change in Y. Fluctuations in A^* offset such spurious changes. For purposes of growth accounting, the production function is better expressed in first differences:

$$\Delta \log (Y_t) = \Delta \log (A_t) + \alpha_t \Delta \log (K_t) + (1 - \alpha_t) \Delta \log (L_t) .$$

Combining that expression with equation (7), one sees that growth of A^* equals growth of true TFP (A) minus an offset for changes in α:

$$\Delta \log (A_t^*) = \Delta \log (A_t) - \Delta \alpha_t \log (K_t / L_t) .$$

The labor input to production is

$$L_t = H_t N_t,$$

where H is hours per worker and N is the number of workers. The capital input takes a similar form:

$$K_t = \bar{k}_t H_t N_t.$$

Hours per worker increase the intensity with which both capital and the workforce are used. With time to build, the number of units of capital M changes slowly, and the intensity of capital usage is proportional to the number of workers N rather to M, unlike in equation (5).

By making the capital input proportional to the logarithmic average of the quality of all capital in service, I implicitly assume that all undepreciated units of capital are used with equal intensity. That differs from much of the putty-clay literature, which assumes that all units of capital above a market-clearing quality are in use while all others are idle.

Substituting for L and K, I re-write the production function (equation 7) as

$$Y_t = A_t^* \bar{k}_t^{\alpha_t} H_t N_t. \tag{8}$$

The firm must face costs of employing more workers than units of capital and of holding hours

[2] Increases in α capture increases in the capital-labor ratio due to a change in technology that increases capital's share of income rather than to a change in the relative price of capital and labor. For example, α increases when businesses use computers and word processing software to do tasks formerly performed by secretaries. The business can generate the same revenue with lower labor costs but with increased outlays for computers and software.

per worker above its "normal" level or it would do so all the time. Those costs are discussed below.

4.2 The Firm's Maximization Problem

The representative firm seeks to maximize the present discounted value (PDV) of future after-tax net receipts. That firm receives revenues net of taxes on production and imports (indirect taxes), corporate taxes, and business transfer payments. The firm's costs are the cost of new capital, labor compensation at "normal" levels of workers per unit of capital and hours per worker, and the costs of deviating workers per unit of capital and hours per worker from normal levels. All of those costs are on an after-tax basis. The firm may also face future gains or losses due to past decisions, such as the tax value of depreciation allowances on existing capital, but those do not affect its current decisions.

Representative firm g receives price p_{gt} for its output Y_{gt}. Following Abel and Eberly (2011), I assume that the inverse demand function for firm g's output is

$$\frac{p_{gt}}{p_t} = \left(\frac{Y_{gt}}{h_{gt}Y_t}\right)^{-\frac{1}{\eta_t}},$$

where h_{gt} is the firm's share of the aggregate demand curve, and $\eta_t > 1$ is a time-varying price elasticity of demand common to all firms. The difference of $1/\eta_t$ from zero reflects the firm's unmeasured intangible capital, such as its reputation, organizational capital, or market power. Prices are net of taxes on production and imports. Because each firm has the same elasticity of demand, $p_{gt} = p_t$ and $Y_{gt} = h_{gt}Y_t$. In the following, in order to simplify the notation, I sum over all firms, so that the variables are aggregates.

The after-tax cost per real dollar of investment ($p\tau_t$) is its price (pi_t) net of taxes, or

$$p\tau_t = pi_t\left[1 - itc_t - u_t z_t\right],$$

where itc_t is the rate of investment tax credit or research tax credit for new capital purchased at time t, u_t is the corporate tax rate, and z_t is the PDV of depreciation allowances per dollar of capital purchased at time t. The relevant tax rate is the one at the time depreciation is taken, so u_t is actually an expected value at time t. The after-tax cost of new capital is exogenous to the representative firm. The after-tax cost of all capital purchased is the after-tax cost times the real quality of new units of capital times the number of units of new capital, or $p\tau_t k_t F_t$.

The cost of labor to the firm gross of taxes and transfers consists of several pieces. The main component is $w_t H_t N_t$, where w is labor compensation per hour when hours per worker,

the number of units of capital per worker, and growth of the number of workers are all at normal levels. The representative firm takes w as exogenous.

There are three additional costs associated with the number of workers, N_t: the cost of increases in the number of workers relative to the number of units of capital, the quadratic cost of deviations between the number of workers and the number of units of capital, and the cost of adjusting the number of workers. The PDV of those costs incurred at time t is

$$\left(\frac{\alpha_t x_t}{1-\alpha_t x_t} w_t \bar{H}_t N_t - \frac{\alpha_t}{1-\alpha_t x_t} w_t \bar{H}_t M_t\right) + \frac{1}{2} c_M w_t H_t \left(\frac{N_t}{M_t} - 1\right)^2 M_t$$
$$+\frac{1}{2} c_N w_t \left(\frac{dN_t/N_t}{dt} - \bar{n}_t\right)^2 H_t N_t,$$

where $\bar{n}_t$ is normal growth of $\bar{N}_t$ (estimated using growth of H-P-filtered $\bar{N}_t$) and c_M and c_N are exogenous to the firm. Because firms face a cost to adjusting the number of workers but not to adjusting hours per worker, firms rely partly on weekly hours to meet shocks to demand.

The variable x_t, which is slightly less than 1, is added because, in a world of growing labor productivity, capital's share of income will generally be lower than its coefficient in production. To see the intuition, consider a new vintage of capital, the labor it uses, and the output it generates. The labor cost associated with the undepreciated capital from that vintage rises roughly with output per worker. The average output generated by all vintages of capital rises at the same rate. However, the output and thus the income generated by the undepreciated capital from a given vintage rises more slowly than that average, because capital per unit of capital from that vintage is fixed but economy-wide capital per worker grows over time. Because labor income from a given vintage of capital grows more rapidly than total income from that vintage, the capital income from a new vintage of capital is front-loaded relative to total income from that vintage. That front-loading means that the ratio of the expected present discounted value (PDV) of capital income to the PDV of total income from a vintage, which the firm sets equal to capital's coefficient in production, exceeds the expected average ratio of capital income to total income from that vintage. (See the appendix on the effect of productivity growth on capital's share of income for a mathematical treatment.)

The PDV of the cost of hours per worker differing from their normal level $\bar{H}_t$ incurred at time t is

$$\frac{1}{1-\alpha_t x_t} w_t \left(H_t - \bar{H}_t\right) N_t + \frac{1}{2} c_H w_t \left(\frac{H_t}{\bar{H}_t} - 1\right)^2 \bar{H}_t N_t.$$

The representative firm takes c_H and $\bar{H}_t$ as exogenous. For $\alpha_t x_t = 1/3$, roughly its estimated

value, the first component corresponds to an overtime premium of 50 percent, a common premium in the United States.

Taking all of those pieces together, the PDV of the cost of labor to the firm gross of taxes and transfers incurred at time t is

$$\frac{1}{1-\alpha_t x_t} w_t H_t N_t - \frac{\alpha_t}{1-\alpha_t x_t} w_t \bar{H}_t M_t + C_t, \text{ where}$$

$$\begin{aligned} C_t &= \frac{1}{2} c_N w_t \left(\frac{dN_t/N_t}{dt} - \bar{n}_t \right)^2 H_t N_t + \frac{1}{2} c_H w_t \left(\frac{H_t}{\bar{H}_t} - 1 \right)^2 \bar{H}_t N_t \\ &\quad + \frac{1}{2} c_M w_t H_t \left(\frac{N_t}{M_t} - 1 \right)^2 M_t. \end{aligned}$$

In long-term equilibrium, in which $H_t = \bar{H}_t$, $N_t = M_t$, and $\frac{dN_t/N_t}{dt} = \bar{n}_t$, and setting $x_t = 1$, that PDV simplifies to $w_t H_t N_t$, the standard pre-tax cost of labor.

Revenues minus labor costs are subject to the corporate income tax and are also the source for business transfer payments. Assuming a tax rate of u and a transfer rate of b, both revenues and labor costs are multiplied by $(1-u_t)(1-b_t)$. To simplify notation, I replace that expression with $(1-ub_t)$.

The PDV of after-tax net receipts also contains two items that I treat as exogenous to the firm's decision at time t. The first of those is the tax deduction for net interest paid by the firm, or $u_t i_t^B B_t$, where i_t^B is the interest rate paid by the firm on its net bonds B_t. For simplicity, I assume that the firm's choice of financing, i.e., its choice of B, is independent of its investment decision. Alternatively, Summers (1981) assumes that firms finance a fraction b of new investment by issuing debt. The second exogenous item is the impact on current and future after-tax net receipts of past decisions, denoted D_t. The primary component of D_t is the PDV of the tax value of depreciation allowances remaining on capital in place at time t. In addition, D_t includes the portion of labor costs C incurred prior to time t but affecting net receipts after time t. With those additions, the PDV of after-tax net receipts is

$$V_t = \int_{j=t}^{\infty} \left\{ \begin{array}{c} (1-ub_j) \left[\begin{array}{c} p_j Y_j - \frac{1}{1-\alpha_j x_j} w_j H_j N_j \\ + \frac{\alpha_j}{1-\alpha_j x_j} w_j \bar{H}_j M_j - C_j \end{array} \right] \\ + u_j i_j^B B_j - p \tau_j k_j F_j \end{array} \right\} \exp\left(-\int_{i=t}^{j} r_i di \right) dj + D_t. \tag{9}$$

where r is the weighted average of the cost of equity and the pre-tax cost of debt.

4.3 The Amount of Capital in New Units of Capital

The derivative of (9) with respect to k_t yields the first-order condition

$$\int_{j=t}^{\infty} (1 - ub_j) \frac{d(p_j Y_j)}{dk_t} \exp\left(-\int_{i=t}^{j} r_i di\right) dj - p\tau_t F_t = 0.$$

After evaluating $\frac{d(p_j Y_j)}{dk_t}$, that equation becomes

$$\int_{j=t}^{\infty} (1 - ub_j) p_j \left(1 - 1/\eta_j\right) \alpha_j \frac{Y_j}{M_j} \exp\left(-\delta (j - t) - \int_{i=t}^{j} r_i di\right) dj = p\tau_t k_t. \tag{10}$$

To make equation (10) more tractable, I follow a simplification commonly used in the neoclassical investment literature and assume that firms expect several variables—ub, η, α, r, and the growth rates of p ($\dot{p}$) and Y/M ($\dot{y}$)—to remain constant after time t. After making those substitutions and rearranging terms, equation (10) becomes

$$k_t = \frac{\alpha_t (1 - ub_t)(1 - 1/\eta_t) p_t Y_t / M_t}{p\tau_t (\delta + r_t - \dot{p}_t - \dot{y}_t)}. \tag{11}$$

To evaluate Y_t/M_t, we shall see below that when firms order capital they expect M_t will equal N_t when the capital is delivered. In addition, I assume that firms expect, when they place orders for new capital, that hours per worker will have returned to its equilibrium level by the time that capital is delivered at time t, i.e., they assume $H_t = \bar{H}_t$. In that case, output per worker Y/N will have reverted to its cyclically adjusted level, $\bar{y}$, where

$$\bar{y}_t = \frac{Y_t}{N_t} \frac{\bar{H}_t}{H_t}. \tag{12}$$

With those substitutions, Y_t/M_t in equation (11) becomes $\bar{y}_t$.

In the style of the neoclassical literature on investment, the resulting equation can also be expressed as

$$k_t = \frac{\alpha_t (1 - 1/\eta_t)(1 - b_t) \bar{y}_t}{v_t}, \tag{13}$$

where

$$v_t = \frac{pi_t}{p_t} (\delta + r_t - \dot{p}_t - \dot{y}_t) \frac{(1 - itc_t - u_t z_t)}{(1 - u_t)}. \tag{14}$$

In the neoclassical literature, v_t is interpreted as the real rental cost of capital. Overall, the quality of new capital depends positively on capital's coefficient in production, cyclically

adjusted output per worker, and its expected growth rate ($\dot{y}$), and negatively on the real price of new capital (pi/p), the depreciation rate, and the real rate of return ($r - \dot{p}$). The quality of new capital also depends on the tax treatment of new capital relative to that of the revenues generated by such capital, i.e., $\frac{(1-itc-uz)}{(1-u)}$. The primary differences between equation (14) and the neoclassical rental cost of capital are the presence of expected growth of output per worker and the use of the expected growth of the output price, rather than the expected growth of the price of capital in calculating the real rate of return.

4.4 Optimal Units of Capital

Starting from equation (9), the Euler equation for hours per worker (H_t) yields

$$\begin{aligned}\left(1-\frac{1}{\eta_t}\right)p_tY_t \quad &\approx \quad \frac{1}{1-\alpha_t x_t}w_tH_tN_t + c_H\left(\frac{H_t}{\bar{H}_t}-1\right)w_tH_tN_t \\ &\quad +\frac{1}{2}c_Nw_t\left(\frac{dN_t/N_t}{dt}-\bar{n}_t\right)^2H_tN_t + \frac{1}{2}c_Mw_tH_t\left(\frac{N_t}{M_t}-1\right)^2M_t.\end{aligned}$$

I assume that the quadratic terms are small enough that they can be ignored, leaving

$$\left(1-\frac{1}{\eta_t}\right)p_tY_t \approx \frac{1}{1-\alpha_t x_t}w_tH_tN_t + c_H\left(\frac{H_t}{\bar{H}_t}-1\right)w_tH_tN_t. \tag{15}$$

Ignoring similar quadratic terms, the Euler equation for M_t yields

$$\frac{\alpha_t}{1-\alpha_t x_t}w_t\bar{H}_t + c_M\left(\frac{N_t}{M_t}-1\right)\frac{w_tH_tN_t}{M_t} \approx \frac{(\delta + r_t - \dot{p}_t - \dot{y}_t)\,p\tau_tk_t}{(1-ub_t)}, \tag{16}$$

where $\dot{w}$ is the growth rate of w, which is roughly equal to $\dot{p}_t + \dot{y}_t$ over long periods.

To determine the number of units of capital the representative firm desires, I substitute for the right-hand side of (16) using equation (11), obtaining

$$\frac{\alpha_t}{1-\alpha_t x_t}w_t\bar{H}_t + c_M\left(\frac{N_t}{M_t}-1\right)\frac{w_tH_tN_t}{M_t} \approx \frac{\alpha_t\,(1-1/\eta_t)\,p_tY_t}{M_t}. \tag{17}$$

To evaluate that equation, I assume that firms expect, when they place orders for new capital, that hours per worker will have returned to its equilibrium level by the time that capital is delivered at time t, i.e., they assume $H_t = \bar{H}_t$. Substituting for the right-hand side of (17) using (15) and making that assumption, I obtain

$$\frac{\alpha_t}{1-\alpha_t x_t}w_t\bar{H}_t + c_M\left(\frac{N_t}{M_t}-1\right)\frac{w_t\bar{H}_tN_t}{M_t} \approx \frac{\alpha_t}{1-\alpha_t x_t}\frac{w_t\bar{H}_tN_t}{M_t}.$$

The solution of that equation is $M_t = N_t$. Firms try to match the number of units of capital to expected employment.

4.5 New Units of Capital

The desired flow of new units of capital, F_t, equals the growth of employment plus replacement demand:

$$F_t = \frac{dN_t}{dt} + \delta M_t.$$

To match the lags in the data between employment growth and investment, it is necessary to modify that equation to incorporate various factors that Kydland and Prescott (1982) call "time to build." This concept captures the phenomenon that investment does not respond immediately to factors determining the number or quality of new units of capital. Time to build is influenced by the time it takes for businesses to gather information and to translate that information into orders for new capital, as well as the time it takes for those orders to be filled. Investment will also lag behind changes in the desired stock if capital is "lumpy," e.g., if businesses do not adjust their capital stock until some threshold level of adjustment is required.[3]

Time to build complicates the relationship between the number of new units of capital and growth of employment in several ways. First, as discussed above, businesses expect that Y/N will have returned to $\bar{y}$ by the time new capital is delivered. Second, because investment is lumpy, firms do not invest in every period, so investment depends on the change in output and expected output since the last investment cycle. Third, firms expect demand to gradually converge to potential output, so investment depends partly on expectations of potential output. Finally, investment occurs gradually between the time capital is ordered and when final delivery takes place. For example, a new structure counts toward investment in every quarter that construction activity occurs.

The assumption that firms expect Y/N to return to $\bar{y}$ implies that the desired units of capital depend on expectations of future $Y/\bar{y}$. To take account of lumpy investment, I assume firms order capital once every T periods.[4] (In this case, the "representative" firm is only representative of firms in a particular investment cycle.) As an approximation, firms target the number of units of capital to equal expected employment mid-way through the order cycle. Assume that a firm places orders at time t' for capital whose delivery will be completed at

[3] For evidence that capital is lumpy, see Doms and Dunne (1998) and Cooper, Haltiwanger, and Power (1999).

[4] A common alternative to the modeling framework used in this paper is to assume quadratic costs of adjusting investment. In my model, that approach proves inconsistent with the data. Quadratic costs of adjusting units of capital imply that recent movements in output should matter the most in determining investment. For structures, however, the change in output in the most recent four quarters has a smaller effect on investment than the change in output in the preceding four quarters.

time t. Denoting time t' expectations of output at time $t+T/2$ as ${}_{t'}Y_{t+T/2}$,

$$M_{t+T/2} = {}_{t'}Y_{t+T/2}/{}_{t'}\bar{y}_{t+T/2}.$$

Firms expect output Y to converge toward potential $\bar{Y}$ according to

$$\frac{{}_{t'}Y_{t+T/2}}{{}_{t'}\bar{Y}_{t+T/2}} - 1 = \psi\left(\frac{Y_{t'}}{\bar{Y}_{t'}} - 1\right). \tag{18}$$

A larger value of ψ means a slower expected convergence of output toward its potential. After some algebra, shown in the appendix, that convergence assumption implies that investment depends on expectations of full employment, $\bar{N}$, defined to equal $\bar{Y}/\bar{y}$. I assume that firms expect $\bar{N}$ to grow at rate $\bar{n}_{t'}$ between time t' and time $t+T/2$.

To match the model to the real world, I assume that the length of investment cycles varies across firms. A fraction ϕ_j of firms have an investment cycle T_j periods long with a corresponding ψ_j. As shown in the appendix, the number of new units of capital delivered in period t, after switching to discrete time, can be expressed as

$$\begin{aligned} F_t \;=\; & \sum_{i=1}^{t-t'}\sum_j \beta_j \frac{d\left[\left(Y_{t-i}/\bar{y}_{t-i}\right)\left(1+\bar{n}_{t-i}\right)^{t-t'+T_j/2}, T_j\right]}{(t-t')\times T_j} \\ & + \sum_{i=1}^{t-t'}\sum_j \gamma_j \frac{d\left[\bar{N}_{t-i}\left(1+\bar{n}_{t-i}\right)^{t-t'+T_j/2}, T_j\right]}{(t-t')\times T_j} + \delta M_{t-1}, \end{aligned} \tag{19}$$

where, to simplify notation, $d[X_j, i]$ denotes $X_j - X_{j-i}$, $\beta_j = \phi_j\psi_j$, $\gamma_j = \phi_j\left(1-\psi_j\right)$, and $\sum_j \beta_j + \sum_j \gamma_j = 1$. That expression shows that new units of capital depend on past growth of both the ratio of output to productivity and full employment. Since the former is determined by demand and the latter by labor supply, investment depends on the growth of both aggregate demand and labor supply. Real investment (I_t) is found by multiplying k_t by F_t.

4.6 Investment, the Market Value of Capital, and Tobin's q

Thus far, I have modeled the amount of capital in a new unit of capital, k, as a function of the cost of capital, and thus of the real rate of return. An alternate strategy is the one used in the Tobin's q literature, which models investment as a function of the market value of existing capital. In order to do this, I use the method of the maximum principle, following in the footsteps of Hayashi (1982).

In equation (9), the expression for the PDV of future after-tax net receipts, D_t is exogenous at time t and B is independent of investment. Consequently, the firm's maximization

problem can be expressed using the Hamiltonian function

$$H_t = \left\{ \begin{array}{c} (1-ub_t) \left[\begin{array}{c} p_j Y_j - \frac{1}{1-\alpha_j x_j} w_j H_j N_j \\ + \frac{\alpha_j}{1-\alpha_j x_j} w_j \bar{H}_j M_j - C_j \end{array} \right] \\ -p\tau_t k_t F_t + \lambda_t \frac{dK_t}{dt} \end{array} \right\} \exp\left(- \int_{i=0}^{t} r_i di \right),$$

in which k_t is the control variable affecting the state variable K_t. λ_t is the shadow price of K_t in generating future after-tax revenues.

After substituting for dK_t/dt using equation (6) and a bit of rearranging, the first order condition $\frac{\partial H_t}{\partial k_t} = 0$ yields

$$k_t = \frac{\lambda_t K_t}{M_t} \frac{1}{p\tau_t}. \tag{20}$$

That is a key result. Previously, I found that the quality of new capital was a function of many factors. Now, it depends only on the shadow value of existing units of capital and the after-tax cost of new capital.

To render equation (20) empirically useful, it is necessary to evaluate $\lambda_t K_t$. Following a method based on Hayashi (1982), a proof in the appendix shows that

$$V_t - V_t^{UF} = \lambda_t K_t + V_t^{UE} + D_t + V_t^W + V_t^B,$$

where V_t^{UE} is the PDV of after-tax profits generated by existing unmeasured intangible capital, V_t^{UF} is the PDV of after-tax profits generated by UIC not yet created, V_t^W is the effect on V_t of capital's share of income being lower than implied by its coefficient in production, and V_t^B is the PDV of the tax deduction for net interest paid, $u_j i_j^B B_j$. Because $V_t^{UE} + D_t + V_t^W + V_t^B$ includes all elements of the firm's market value except for the value of existing capital in future production, I can conclude that $\lambda_t K_t$ is the market value of existing capital in future production. Consequently, the amount of capital in new units of capital, k_t, depends on the market value of existing capital and the after-tax cost of new capital.

Using that representation of k_t, investment can be expressed as a function of Tobin's q. K_t^N, the neoclassical stock of capital calculated using the perpetual inventory method, is the real pre-tax "replacement" cost of existing capital as defined in calculating Tobin's q. Dividing both sides of equation (20) by $p_t K_t^N$, the nominal replacement cost, and multiplying through by F_t, the ratio of investment to replacement cost can be expressed as

$$\frac{I_t}{K_t^N} = \frac{\lambda_t K_t}{p_t K_t^N} \frac{1}{1 - itc_t - u_t z_t} \frac{F_t}{M_t}.$$

The expression $\frac{\lambda_t K_t}{p_t K_t^N}$ is Tobin's q, so

$$\frac{I_t}{K_t^N} = \frac{q_t}{1 - itc_t - u_t z_t} \frac{F_t}{M_t}. \tag{21}$$

The ratio of investment to replacement cost equals Tobin's q, adjusted for the tax treatment of new capital, times a function of replacement demand, aggregate demand, and labor supply.

5 The Properties of Investment Compared with Those in Other Models

While many of the properties of investment are the same in the CPC model as in standard models, there are some critical differences. To simplify the comparisons, I assume perfect competition ($\eta = \infty$), no capital transfers ($b = 0$), and no time to build. The exception is the Tobin's q model, in which the desired rate of investment would be infinite if there were no adjustment costs.

5.1 Comparison with the Tobin's q Model

Recall that in the traditional q model of investment

$$\frac{I}{K} = h\left(\frac{q}{1 - itc - uz}\right).$$

With CPC capital, absent time to build, equations (19) and (21) become

$$\frac{I}{K} = \frac{q}{1 - itc - uz}\left(\frac{dY/Y}{dt} - \frac{dy/y}{dt} + \delta\right).$$

The key similarity between the traditional Tobin's q model and the CPC model is that investment depends in both on tax-adjusted q. Investment depends positively on asset prices and negatively on the after-tax price of new capital.

The primary difference between the q and CPC models is that investment depends only on tax-adjusted q in the former but also on the growth of aggregate demand less the growth of productivity in the latter. In the Tobin's q model, a firm's desire to invest in new capacity is undiminished by low capacity utilization, as long as stock prices are high. In the CPC model, however, investment depends on how rapidly output has been growing, and thus on current utilization. From the perspective of the CPC model, the importance of cash flow in empirical estimates of the Tobin's q model reflects an omitted variable (growth of demand) rather than liquidity constraints.

That difference between the two models has an important implication for policy. In the

CPC model, policies to stimulate investment by changing after-tax q are less effective in a recession, when aggregate demand is falling, than in a recovery, when aggregate demand is growing. In the traditional Tobin's q model, however, the impact of such policies on investment is independent of the state of the economy. Examples of such policies are a reduction in interest rates in order to boost q or a change in the tax treatment of depreciation in order to boost z.

5.2 Comparison with the Neoclassical Model

Absent time to build, real investment in the neoclassical model can be written as

$$I = \alpha \frac{Y}{v} \left(\frac{dY/Y}{dt} - \frac{dv/v}{dt} \right) + \delta K.$$

Combining equations (13) and (19), investment in the CPC model is

$$I = \alpha \frac{Y}{v} \left(\frac{dY/Y}{dt} - \frac{dy/y}{dt} + \delta \right).$$

There are many similarities between the two models. A key similarity is that, unlike the Tobin's q model, both the neoclassical and CPC models find that investment depends positively on both the level and growth rate of output. The response of the level of investment to the growth of output is known as the "accelerator." In addition, investment in both the neoclassical and CPC models is proportional to capital's coefficient in production and is positively related to replacement demand. Finally, through the rental cost of capital v, real investment in both models is negatively related to the real after-tax price of investment and the combined cost of debt and equity (r).

The key difference between the neoclassical and CPC models is how investment responds to the cost of capital. In the neoclassical model, investment responds to both the level and the growth rate of the cost of capital. However, in the CPC model, investment responds only to the level of the cost of capital. Unlike in the neoclassical model, investment does not behave symmetrically to output and to the cost of capital in the CPC model. In the CPC model, firms can change the number of units of capital in response to higher output but cannot change the quality of existing units of capital in response to a change in the cost of capital. However, firms are free to do both in the neoclassical world. As discussed above, empirical work is more consistent with the CPC model.

Another way to express that difference is that investment in the neoclassical model attempts to restore the actual capital stock K to the desired stock K^*, whereas investment in the CPC model attempts to restore the economy-wide utilization of capital to its desired rate.

An increase in output boosts both K^* and capacity utilization, and so has a similar impact on investment in both models. However, a reduction in the cost of capital boosts K^*, leading to higher investment in the neoclassical model, but has no effect on capacity utilization. A lower cost of capital affects investment in the CPC model only by increasing the quality of new units of capital.

Investment also responds in different ways to a productivity shock in the two models. Consider a 1 percent shock to both output Y and output per worker y. In both models, Y/v increases, boosting investment. The accelerator gives a large additional boost to investment in the neoclassical model. In the CPC model, however, the accelerator is offset by faster growth of productivity. Because of increased productivity, the existing capital stock can meet the increase in demand, so there is no need for firms to purchase additional units of capital. In fact, in the CPC model an increase in productivity not matched by higher output reduces investment. That could explain Basu, Fernald, and Kimball's (2006) finding that improved technology causes nonresidential investment to fall in the short run.

There are several other differences between the two models. In the neoclassical model, replacement demand equals the (real) dollar value of depreciating capital. In the CPC model, however, new units are likely to be of higher quality than the units they replace, so replacement demand is a larger share of investment. In addition, investment depends positively on expected future growth of productivity in the CPC model but is independent of expected future growth of productivity in the neoclassical model. With CPC capital, increased future productivity raises the PDV of revenue produced by an extra dollar of capital. In the neoclassical model, however, the marginal product of capital in the future always equals the price of new capital, so additional future productivity leaves the PDV of an extra dollar of new capital unchanged. In the cost of capital, $\dot{p}$ is the expected growth of the price of output in the CPC model but is the expected growth of capital prices in the neoclassical model. An increase in the output price raises the value of output produced by a unit of CPC capital but has no impact on the value of output produced by a dollar of neoclassical capital, which is instead determined by the price of new capital.

5.3 Comparison with Standard Putty-Clay Models

Because there is no set of assumptions common to all putty-clay models, there are many specifications for investment. Possibly the best-known empirical specification is that of Bischoff (1971), which in the absence of lags simplifies to

$$I = \alpha\chi\frac{Y}{v}\left(\frac{dY/Y}{dt} + \delta\right),$$

where χ is a simple suboptimal rule of thumb used because the productivity of new capital is not observable. In this model, in contrast to the CPC model, productivity growth does not subtract from the accelerator. Productivity growth is also absent from the investment equations of Macroeconomic Advisers (2008). Unlike in the CPC model, expected future productivity growth does not affect investment in the Bischoff and Macroeconomic Advisers models. In both respects, the standard putty-clay models make the same assumptions as the neoclassical model.

6 Refinements of the Basic Model

To better reflect conditions in the real world, I modify the basic model in several ways. In the real world, there are many types of nonresidential capital.[5] Several of those types—capital specific to mining and farming, inventories, and land—require special treatment. In addition, the response of firms to taxes may differ from the theoretical response for various reasons, requiring modifications in order to estimate the model.

6.1 Treatment of Mining and Farming

Mining is unusually capital-intensive. The ratio of the net stock of capital to value added in mining averaged 5.3 in 1987-2011, compared with 1.6 in nonfarm business. In the context of the model, capital's coefficient in production α is much higher for mining than for other sectors. Excluding capital specific to mining (mining and oilfield machinery and mining exploration, shafts, and wells), the ratio of capital to value added averaged 1.6 in mining in 1987-2011, the same as in nonfarm business, so mining-specific capital accounts for all of the excess α in mining.

According to equation (13), an increase in α leads to an increase in the quality of new units of capital. However, we would expect an increase in α due to an increase in the mining sector to boost capital per worker only in that sector. One possible solution is a two-sector model, but that would greatly complicate the estimation.

Instead, I treat increases in α due to disproportionate increases in mining-specific investment as equilibrium increases in the ratio of units of capital to workers, or M/N. In equation (9) for V_t, M_j becomes M_j/S_j, where S_j is the equilibrium number of units of capital per worker. To normalize S, I assume that

$$S_t = 1 + s_{min,t},$$

[5] In addition, there are several types of residential capital, including some owned by business (rental properties). I do not attempt to model residential investment in this paper.

where $s_{min,t}$ is the equilibrium ratio of units of mining-specific capital to workers at time t. Thus, the equilibrium ratio of non-mining-specific units of capital to workers is 1.

With those changes, quality of new units of type-m capital as a function of the cost of capital is

$$k_{mt} = \frac{(\alpha_t/S_t)(1-1/\eta_t)(1-b_t)\bar{y}_t}{v_{mt}},$$

where variables with an m subscript are specific to type-m capital. The expression for k_{mt} as a function of the market value of capital, as in equation (20), is unchanged, except that market value per unit of capital and the tax treatment of new capital are those for type-m capital.

In this paper, I do not analyze investment specific to mining and farming. The primary reason for that exclusion is that the CPC model does not do a good job of explaining those sectors. Investment in the CPC model depends primarily on growth of demand, while investment in those sectors responds strongly to the price of their output.

6.2 New Units of Capital

Let the parameter σ_{mt} represent type-m capital's share of total units of nonfarm non-mining capital delivered at time t in the absence of cyclical effects. As shown in the appendix, with many types of capital equation (19) for deliveries of new units of capital at time t becomes

$$\begin{aligned} F_{mt} &= \sigma_{mt} \sum_{i=1}^{t-t'_m} \sum_j \beta_{mj} \frac{d\left[(Y_{t-i}/\bar{y}_{t-i})(1+\bar{n}_{t-i})^{t-t'_m+T_{mj}/2}, T_{mj}\right]}{(t-t'_m)\times T_{mj}} \\ &\quad + \sigma_{mt} \sum_{i=1}^{t-t'_m} \sum_k \gamma_{mk} \frac{d\left[\bar{N}_{t-i}(1+\bar{n}_{t-i})^{t-t'_m+T_{mj}/2}, T_{mk}\right]}{(t-t'_m)\times T_{mk}} \\ &\quad + \sigma_{mt} \sum_{m \notin m\&f} \delta_m M_{m,t-1}, \end{aligned} \tag{22}$$

where $\sum_{mj} \beta_{mj} + \sum_{mk} \gamma_{mk} = 1$. The notation $m \notin m\&f$ denotes nonfarm non-mining types of capital. The length of the investment cycle T_{mj} can differ by type of capital, and so has a subscript m. As in the case of a single type of capital, investment depends on the growth of both aggregate demand and labor supply. Estimated coefficients on the γ_{mk} for shorter T_{mk} are generally negative, so I use different lengths of investment cycles for demand and supply.

6.3 Units of Inventory Capital

The stock of inventories displays the same putty-clay characteristics as the stock of fixed capital, with short-run variations being dominated by short-run variations in output. Year-

over-year growth of a five-quarter moving average of real nonfarm business output explains 70 percent of the variance of year-over-year growth of real nonfarm inventories in 1960-2013 (see Figure 2).

However, inventories themselves are not held long enough to be considered as "clay." Manufacturers, retailers, and merchant wholesalers have on average held an item in inventory for less than two months since the data begin in 1980. For inventories to behave as if they are putty-clay, it must be that inventory technologies, rather than the inventories themselves, are putty-clay. For example, the amount of inventories held depends on the type of store a retailer has, a wholesaler's distribution system, or a manufacturer's use of just-in-time inventories. In each case, a business will hold more inventories the higher its sales, but will not be willing to incur the cost of changing its way of doing business in response to short-term movements in real rates of return. The inventory analogue to a unit of capital is a unit of inventory technology.

6.4 Cost of Capital for Inventories and Land

Inventories and land require different treatment from business fixed capital because they do not depreciate, so their net cost depends partly on their resale value. In the case of inventories, k_{it} is the amount of real inventory in a new unit of inventory technology, F_{it} is new units of inventory technology, and δ_i is the rate at which units of inventory technology are retired. I assume that the amount of inventories associated with a unit of inventory technology grows with output per worker, at an expected rate of $\dot{y}_t$. That assumption creates a symmetry with fixed capital; as an increase in output per worker does not increase the number of units of fixed capital, so an increase in real inventories stemming from higher productivity does not increase units of inventory technology. The value of net sales of inventories from existing units of inventory technology are thus a fraction $\delta_i - \dot{y}_t$ of the existing stock of inventories. The price of inventories is expected to increase at rate $\dot{p}_{it}$, so nominal resales from a unit of inventory technology decrease at rate $\delta_i - \dot{y}_t - \dot{p}_{it}$. Discounting those future resales at nominal rate r, the after-tax PDV of future resales from a new unit of inventory technology is

$$p_{it}k_{it}\int_{j=t}^{\infty}(1-u_j)\left(\delta_i-\dot{y}_j\right)\exp\left(-\left[r_j+\delta_i-\dot{y}_j-\dot{p}_{ij}\right]\right)dj.$$

As for fixed capital, I assume that businesses expect r, $\dot{y}$, and $\dot{p}_i$ to be constant in the future and expect the tax rate to be u_t. The PDV of future resales for a unit of new inventory technology is

$$(1-u_t)\,p_{it}k_{it}\frac{\delta_i-\dot{y}_t}{r_t+\delta_i-\dot{y}_t-\dot{p}_{it}}.$$

Subtracting the PDV of resales from the gross after-tax cost of a new unit of inventory technology, $(1-u_t)\,p_{it}k_{it}$, I find that the expected net after-tax cost of a new unit of inventory technology is

$$p\tau_{it} = (1-u_t)\,p_{it}\frac{r_t - \dot{p}_{it}}{r_t + \delta_i - \dot{y}_t - \dot{p}_{it}}. \tag{23}$$

The net cost of land differs in three ways from the net cost of inventories. First, the depreciation rate is that of the associated structure. Second, the quality of a unit of land does not increase with productivity, so the $\dot{y}_t$ terms disappear. Third, land is not depreciable for tax purposes and if a business reuses land after the associated structure depreciates, it does not pay capital gains tax, so there are no tax terms. The cost of land associated with a structure of type m, net of the PDV of its resale value, is thus

$$p\tau_{L,m,t} = p_{L,t}\frac{r_t - \dot{p}_{L,t}}{r_t + \delta_m - \dot{p}_{L,t}},$$

where the subscript L denotes land. The appendix shows how that expression is used to calculate the number of units of land.

6.5 Responses to Taxes Differing from Theory

There are various reasons that investment may not respond to tax parameters as strongly as the model predicts. Firms may not expect tax rates to remain at current levels during the entire service life of new capital or during the period when depreciation is taken. In addition, firms are assumed to adopt the method of depreciation that maximizes the PDV of allowances. However, Knittel (2007) finds that many firms failed to use bonus depreciation in 2002-2004. Also, all investment is assumed to be subject to depreciation, as in the corporate tax code, but investment by noncorporate businesses and nonprofit institutions is not.

On the other hand, my use of corporate tax law to calculate the tax parameters may miss the effects of changes in tax law for individuals. For example, there was a large increase in partnerships' share of investment in structures in the 1980s, at least in part because of more favorable treatment of passive losses for individuals.

To capture those impacts, I assume that firms expect the current tax rate to persist with probability $c1$ and expect the rate to return to its sample average with probability $1-c1$. When they invest, firms expect to use current law to depreciate share $c2$ of new investment and to use sample-average tax law to depreciate the rest.

With those assumptions, $1 - itc_{mt} - u_t z_{mt}$ in equation (14) and implicit in equation (20)

becomes

$$G_{mt} = \left(\begin{array}{c} (1 - itc_{mt} - u_t z_{mt})^{c1c2} (1 - itc_{mt} - \hat{u} z_{mt})^{(1-c1)c2} \times \\ \left(1 - \widehat{itc}_m - u_t \hat{z}_m\right)^{(1-c2)c1} \left(1 - \widehat{itc}_m - \hat{u} \hat{z}_m\right)^{(1-c1)(1-c2)} \end{array} \right), \tag{24}$$

where $\hat{u}$, $\widehat{itc}_m$, and $\hat{z}_m$ denote sample averages. I use sample averages so that the estimates of $c1$ and $c2$ do not affect the average level of v_{mt}. For structures, G_{mt} is divided by the expression $1 + c3\ D_{80s}$, where D_{80s} is a dummy variable for 1981-1986.

7 Background of the Estimation

A primary innovation of this paper is the use of investment to identify the shares of income and of firms' market value contributed by various types of capital. I do that using two iterative steps. In the first step, I estimate investment equations for measurable capital. Those equations use a Kalman filter to estimate unobservable $(1 - 1/\eta_t)\, \alpha_t / S_t$ (denoted α_t^*). In the second step, I distribute total market value among the various types of capital and recalculate S_t and the σ_{mt} using the α_t^* and other parameters estimated in the first step. Those estimates of S_t, σ_{mt}, and market value by type of capital then become an input into the next iteration of the first step. The procedure must be done iteratively because market value is a nonlinear function of α_t^*, and the right-hand side variables of an equation must be linear functions of α_t^* in order to estimate α_t^* using a Kalman filter.

In this section, I show how total market value is distributed among the various types of capital and presents the investment equations to be estimated. I also discuss a modification of geometric depreciation better suited to the putty-clay nature of capital and present the final form of the investment equations. In addition, I discuss the parameters estimated and the data used in the estimation.

7.1 How the Market Value of Capital Is Estimated

Although the market value of all capital is observable, the allocation of that total among different types of capital is unobservable. To make that allocation, I express the market value of different types of capital in terms of observable variables and parameters estimated in the investment equations.

To estimate the market value of measured capital, I assume that investors have the same expectations of future tax rates as businesses, and so replace $(1 - u_t)$ in equation (11) with $(1 - u_t)^{c1} (1 - \hat{u})^{1-c1}$. In addition, I assume that investors expect b_t to average its H-P filtered value, $\bar{b}_t$, in the future. Combining equations (11) and (20) with those substitutions

and further substituting $\bar{y}_t$ for Y_t/M_t, the market value of capital in future production is

$$V_{mt} = \lambda_{mt} K_{mt} = M_{mt} \frac{\alpha_t^* (1-u_t)^{c1} (1-\hat{u})^{1-c1} \left(1-\bar{b}_t\right) p_t \bar{y}_t}{\delta_m + rpy_t}, \tag{25}$$

where rpy_t is $r_t - \dot{p}_t - \dot{y}_t$. For inventories and land, V_{mt} also includes the PDV of their resale value.

Because the data for V_{mt} is only for nonfinancial corporations, I multiply the right-hand side of equation (25) by the nonfinancial share of the stock of type-m capital. I thus implicitly assume that the other parameters in equation (25) are the same for nonfinancial corporations and for all businesses using nonresidential fixed capital.

Because $1/\eta_t$ differs from zero due to a firm's unmeasured intangible capital, the income from that capital, net of taxes and business transfer payments, is $(1-ub_t)\frac{1}{\eta_t} p_t Y_t$. Ignoring the final term in equation (15) and rearranging, that income is

$$(1-ub_t)\frac{1}{\eta_t} p_t Y_t = (1-ub_t) p_t Y_t \left[1 - \frac{w_t N_t H_t}{p_t Y_t} - \alpha_t^* S_t x_t\right]. \tag{26}$$

To estimate the market value of existing UIC, I assume that such capital depreciates at rate δ_I. In addition, I assume the number of representative firms is constant and investors expect the revenue from a unit of its unmeasurable capital to grow with the output of the firm. Thus, investors expect real income of a unit of undepreciated intangible capital to grow at rate $\dot{y}_t + \hat{n}$, where $\hat{n}$ is the sample-average growth rate of N because UIC is relatively long-lived. As an approximation, I assume that investors ignore cyclical variations of output and labor's share in calculating the market value of existing UIC. Investors also make the same assumptions about u and b used to value tangible capital. With those assumptions, the market value of existing unmeasured intangible capital is

$$V_t^{UE} = (1-u_t)^{c1} (1-\hat{u})^{1-c1} \left(1-\bar{b}_t\right) p_t \bar{Y}_t \frac{1 - \overline{ws}_t - \alpha_t^* S_t x_t}{\delta_I + rpy_t - \hat{n}}, \tag{27}$$

where $\overline{ws}_t$ is labor's share of output at full employment. I estimate $\overline{ws}_t$ by applying an H-P filter to $\frac{w_t N_t H_t}{p_t Y_t}$.

The final step in determining V_{mt} is choosing a value of rpy_t that makes all of the components of firms' market value add up to the total. To do that, first define $\hat{V}_{mt}$ as the hypothetical market value of type-m capital if rpy_t were equal to $\widehat{rpy}$, the sample average of rpy_t. Then

$$V_{mt} \approx \hat{V}_{mt} + \left(rpy_t - \widehat{rpy}\right) \frac{d\hat{V}_{mt}}{d\widehat{rpy}}.$$

That equation applies to every component of the firm's market value, including V_t^{UE} and unused depreciation allowances (in D_t), since the market value of those components depends on rpy_t. Denoting each of those components with the subscript i, summing, and rearranging, yields

$$rpy_t - \widehat{rpy} \approx \frac{\sum_i V_{it} - \sum_i \hat{V}_{it}}{\sum\limits_i \frac{d\hat{V}_{it}}{d\widehat{rpy}}}.$$

Substituting that expression for $rpy_t - \widehat{rpy}$ into the previous equation yields

$$V_{mt} \approx \hat{V}_{mt} + \frac{\sum_i V_{it} - \sum_i \hat{V}_{it}}{\sum\limits_i \frac{d\hat{V}_{it}}{d\widehat{rpy}}} \frac{d\hat{V}_{mt}}{d\widehat{rpy}}. \tag{28}$$

Note that an estimate of rpy_t is not required to solve equation (28). Such an estimate can be backed out using the previous equation, but it may be a poor approximation of the true rpy_t the further one gets from $\widehat{rpy}$.

There are two ways to choose $\widehat{rpy}$, although one of them leads to an implausible result. One way is to solve for $\widehat{rpy}$ such that $\sum_i V_{it} - \sum_i \hat{V}_{it}$, normalized by $p_t\bar{Y}_t$, averages zero over the sample period. That results in an implausibly large value for $\widehat{rpy}$, even after the modification to geometric depreciation discussed below. Instead, I assume a value of 0.04 (or 4 percent) for $\widehat{rpy}$. Since actual $\dot{y}$ averages 0.016 over the sample period, that implies that $r_t - \dot{p}_t$ averages 5.6 percent. So that V_{mt} averages roughly to $\hat{V}_{mt}$, I subtract $c_V p_t \bar{Y}_t$ from $\sum_i V_{it} - \sum_i \hat{V}_{it}$ in equation (28), where c_V is the sample mean of $\frac{\sum_i V_{it} - \sum_i \hat{V}_{it}}{p_t \bar{Y}_t}$.

7.2 Modification to Geometric Depreciation

Geometric depreciation, used to develop the Tobin's q model, may not describe existing putty-clay capital well. With geometric depreciation, the expected remaining service life of existing capital is the same as that of new capital, $1/\delta_m$, so the depreciation rate used to determine the market value of existing capital is the same as that for new capital, δ_m. If units of capital have a fixed service life when new, however, then on average the expected remaining service life of existing capital is half that of new capital, $1/(2\delta_m)$, and the depreciation rate needed to determine market value is approximately $2\delta_m$. Consequently, I use depreciation rates of $1.5\delta_m$ and $1.5\delta_I$ to estimate the V_{mt} and V_t^{UE}.

7.3 Final Form of the Investment Equations

There are a number of reasons that the response of the quality of new units of capital k_{mt} to the market value of capital V_{mt} might be less than theory predicts. Managers may

have different perceptions than investors have about the elements of rpy: the appropriate discount rate, expected price inflation, or expected productivity growth. My model or the data I use may omit or mistreat components of the market value of nonfinancial corporations, which I use in the estimation. One particular issue is that market value includes the value of net overseas capital but I am modeling domestic investment. Another reason for a small response of k_{mt} to V_{mt} is that nonprofit institutions, financial corporations, and noncorporate businesses may not attach the same value to a unit of capital as do nonfinancial corporations. In addition, linearized responses of the quality of capital to rpy may not work well for large deviations of market value from normal, as in the late 1990s. Finally, the modification I make to geometric depreciation may not fully resolve that issue.

To account for all those factors, I multiply the second term on the right-hand side of equation (28) by the parameter $c4$ when estimating k_{mt}, where $c4$ is expected to be between 0 and 1. After making that change to equation (28), using equation (25) to calculate $\hat{V}_{mt}/M_{mt}$, and using equation (24) to evaluate $p\tau_{mt}$, equation (20) becomes

$$pi_{mt}k_{mt} = \frac{1}{G_{mt}}\left[\frac{\alpha_t^*\left(1-\bar{b}_t\right)\left(1-u_t\right)^{c1}\left(1-\hat{u}\right)^{1-c1}p_t\bar{y}_t}{\delta_m+\widehat{rpy}} + c4\,\frac{Vdf_{mt}}{M_{mt}}\right] \tag{29}$$

$$\text{where } Vdf_{mt} \equiv \frac{\sum_i V_{it} - \sum_i \hat{V}_{it} - c_V p_t \bar{Y}_t}{\sum\limits_i \frac{d\hat{V}_{it}}{d\widehat{rpy}}}\frac{d\hat{V}_{mt}}{d\widehat{rpy}}. \tag{30}$$

I estimate four investment equations: one each for equipment, structures, IPP, and inventories. The Bureau of Economic Analysis (BEA) provides estimates of investment and the capital stock for 23 types of equipment, 17 types of structures, and 18 types of IPP, not counting types specific to mining and farming.[6] Investment for each broad category of fixed capital is modeled as the sum of the product of nominal quality (from equation (29)) and the number of new units (from equation (22)) for each type of capital within that category. I divide through by the Congressional Budget Office's (2014a) estimate of nominal potential GDP ($\overline{gdp}_t$) as a correction for heteroskedasticity. The determinants of F_{mt} other than the σ_{mt} are assumed to be the same for each type of capital within a broader category, while many of the determinants of k_{mt} can differ. Thus, for example, the equation for equipment investment (denoted by E) is

$$\frac{\sum_{m\in E} pi_{mt}I_{mt}}{\overline{gdp}_t} = \frac{\sum_{m\in E}\sigma_{mt}pi_{mt}k_{mt}}{\overline{gdp}_t}F_{Et} + \epsilon_{Et},$$

[6] Because a capital stock is necessary to calculate market value, I do not estimate investment for types of capital for which there is no capital stock data: sale of equipment scrap excluding autos; brokers' commissions on sale of structures; and net purchases of used structures.

where $pi_{mt}k_{mt}$ is given by equation (29),

$$G_{mt} = \left(\begin{array}{c} (1 - itc_{mt} - u_t z_{mt})^{c1c2} (1 - itc_{mt} - \hat{u} z_{mt})^{(1-c1)c2} \times \\ \left(1 - \widehat{itc}_m - u_t \hat{z}_m\right)^{(1-c2)c1} \left(1 - \widehat{itc}_m - \hat{u}\hat{z}_m\right)^{(1-c1)(1-c2)} \end{array} \right),$$

$$\begin{aligned} F_{Et} &= \sum_{i=1}^{t-t'_E} \sum_j \beta_{Ej} \frac{d\left[(Y_{t-i}/\bar{y}_{t-i})(1+\bar{n}_{t-i})^{t-t'_E+T_{Ej}/2}, T_{Ej}\right]}{(t-t'_E) \times T_{Ej}} \\ &\quad + \sum_{i=1}^{t-t'_E} \sum_k \gamma_{Ek} \frac{d\left[\bar{N}_{t-i}(1+\bar{n}_{t-i})^{t-t'_E+T_{Ej}/2}, T_{Ek}\right]}{(t-t'_E) \times T_{Ek}} + \sum_{m \notin m\&f} \delta_m M_{m,t-1}, \end{aligned}$$

and ϵ_{Et} is an error term. For structures, G_{mt} is divided by the expression $1 + c3\ D_{80s}$, as discussed above.

The equation for inventory investment differs in several ways from the equations for fixed capital. Inventory investment is the inventories in new units of inventory technology minus the inventories in depreciating units plus the increase in inventories in existing units due to increased output per worker. Assuming that the response to output per worker takes place gradually over five quarters, nominal inventory investment is

$$p_{it}I_{it} = p_{it}k_{it}F_{it} - \delta_i p_{it}K_{it}^N + \frac{\bar{y}_t/\bar{y}_{t-5} - 1}{5} p_{it}K_{it}^N, \tag{31}$$

where K_{it}^N is real inventories. Using equation (23) to substitute for $p\tau_{mt}$ in equation (11), the inventory analogue for equation (29) is

$$p_{it}k_{it} = \frac{1}{1-u_t}\left[\frac{\alpha_t^*\left(1-\bar{b}_t\right)(1-u_t)^{c1}(1-\hat{u})^{1-c1} p_t\bar{y}_t}{\delta_i + \widehat{rpy}} \frac{\delta_i + \widehat{rpy} - \dot{p}_{it}^r}{\widehat{rpy} + \dot{y}_t - \dot{p}_{it}^r} + c4\ \frac{Vdf_{it}}{M_{it}}\right],$$

where $\dot{p}_{it}^r$ is expected growth of p_{it}/p_t, assumed to equal its sample average. Assuming that the investment cycle for inventories is fairly short, the number of new units of inventory technology does not depend on $\bar{N}$ and can thus be expressed as

$$F_{it} = \sigma_{it}\left\{\beta_i \Delta\left(\frac{Y_t}{\bar{y}_t}\right) + (1-\beta_i)\frac{d\left[\frac{Y_{t-1}}{\bar{y}_{t-1}}(1+\bar{n}_{t-1})^2, 4\right]}{4} + \sum_{m \notin m\&f} \delta_m M_{m,t-1}\right\}.$$

Making those substitutions for $p_{it}k_{it}$ and F_{it} in equation (31), dividing through by $\overline{gdp}_t$ and adding an error term yields the equation used to estimate nonfarm inventory investment.

7.4 Parameters

To estimate the common factor α_t^* using a Kalman filter, the investment equations are estimated as a system using maximum likelihood. The standard error for the quarterly innovation to α_t^* is set to 0.0005, or about 0.05 percent of GDP. To account for time to build in the effect of the market value of capital on investment, two-quarter moving averages are used for the Vdf_{mt} for equipment and six-quarter moving averages are used for the Vdf_{mt} structures and intellectual property products. The same length moving averages are used for the G_{mt} except for equipment, for which I use only the contemporaneous value.

The coefficients $c1$, $c2$, and $c4$ are assumed to be the same across all equations. For equipment, I use investment cycles of 4, 8, 16, and 24 quarters for T_{Ej} and of 8 and 24 quarters for T_{Ek}; all have positive coefficients. For structures and IPP, I eliminate investment cycles with negative estimated coefficients. As a result, the T_{mj} are 8, 16, and 24 quarters for structures and 4, 8, and 16 quarters for IPP, and only a T_{mk} of 24 quarters is used for structures and IPP. Time to delivery, $t - t'_m$, is two quarters for equipment and four quarters for structures and intellectual property products.

To calculate the σ_{mt}, the errors in the investment equations are assumed to come from the F_{mt}. Thus, F_{mt} can be calculated by dividing I_{mt} by estimated k_{mt}. Raw values for the σ_{mt} can then be found by inverting equation (22). The σ_{mt} values used are H-P-filtered values of those raw values. To estimate S_t for use in calculating V_t^{UE}, I calculate the stock of units of each type of capital using a perpetual inventory method and divide the total number of units of capital by the number of non-mining units. To avoid perpetuating errors in the F_{mt} through their effect on future replacement demand, replacement demand is calculated by constraining the M_{mt} to sum to the number of units that would obtain if there were no errors in the F_{mt}.

7.5 Data

The sectoral coverage of investment and the variables used to estimate it should be consistent. Thus, output is the gross value added by sectors using private nonresidential capital: nonfarm business less tenant-occupied housing plus nonprofit institutions. (Tenant-occupied housing is part of nonfarm business output but does not use nonresidential capital.) Nominal output excludes taxes on production and imports, which are not part of the net revenue of the firm. Employment is the sum of employment in the nonfarm business and nonprofit institution sectors from the Bureau of Labor Statistics. Hours per worker are found by dividing total hours worked in nonfarm business and nonprofit institutions by employment. Estimates of $\bar{N}_t$, $\bar{H}_t$, and $\bar{y}_t$ are discussed in the appendix.

The market value of the nonfarm nonfinancial corporate sector is calculated using data from the Federal Reserve Board's flow of funds. Following Hall (2001), the value of capital equals the value of debt and equity less the value of financial assets. Debt and assets are valued at face value in the flow of funds, so I convert them to market values as shown in the appendix. The appendix also provides details on how variables related to taxation are constructed. Data are quarterly, and the equations are estimated from the first quarter of 1960 through the fourth quarter of 2013.

8 Empirical Results and Their Implications

8.1 Empirical Results

The model fits the investment data quite well, not surprising given that α^* is estimated using a Kalman filter (see Figure 3). To some extent, variations in α^* capture serially-correlated errors in the investment equations. Serial correlation is less important for inventory investment, which tends to be quite volatile from quarter to quarter, so the fit is not as good (see Figure 4).

The first column of Table 1 shows the estimated coefficients when α^* is estimated using a Kalman filter. I constrained the coefficient on the corporate tax rate, $c1$, to 1.0 because it is greater than 1.0 when freely estimated. The estimated coefficient on the tax treatment of new capital, $c2$, is also close to 1, and the boost to structures investment during the 1980s, as measured by $c3$, is statistically significant. Although $c4$, the estimated coefficient on Tobin's q, is well below 1.0, it is highly significant, with a z-statistic of almost 16.

The pattern of the $\sum_j \beta_{mj}$, the sum of coefficients on changes in demand, is surprising. A priori, one would expect changes in demand to have a greater weight and labor supply a smaller weight the shorter is the time to build. However, changes in demand have a greater impact on structures than on equipment or IPP. One possible reason is that using the same σ_{mt} for replacement demand as for changes in units understates the relative importance of replacement demand for shorter-lived capital. That could boost the weight on labor supply, which like replacement demand is fairly stable, reducing the relative weight on changes in demand.

The model fits the data less well when α^* is constrained to be constant over the sample (see Figure 5). Even so, the model fits well after 1990 and does a good job at identifying turning points. (I constrain $c3$ to have the same value as when α^* is estimated using a Kalman filter because it is an implausibly large 0.59 when unconstrained.) Estimated coefficients, shown in the third column of Table 1, are similar to those when α^* is estimated using a Kalman filter.

The coefficients on the tax treatment of new capital and on Tobin's q are smaller, while the pattern of the $\sum_j \beta_{mj}$ is somewhat less surprising.

Optimal lags on the factors determining new units of capital are much longer than on the factors determining the quality of new units of capital, consistent with lumpy investment. Under lumpy investment, the number of new units of capital depends on the change in the desired number since the last investment was made, but the quality of new units of capital depends on market values and tax rates when plans for new investment were made.

8.2 What Drives Investment?

The accelerator—changes in the ratio of output to cyclically adjusted output per worker—dominate short-run movements in investment (see Figure 6). Variations in the growth of output less worker productivity directly accounted for 61 percent of variations in the year-over-year growth of real private nonresidential fixed investment during the sample period. That is despite the latter including mining and farming investment, for which no accelerator impact was estimated.

In contrast, changes in q have had a relatively small direct impact, accounting for 10 percent of the variation in year-over-year growth of real investment (see Figure 7). However, that likely understates the importance of financial shocks for business investment for a few reasons. First, monetary policy is often used to counter shocks to the financial system, putting downward pressure on the cost of debt after lower stock prices have boosted the cost of equity. Second, accelerator effects on investment include the impacts of changes in output that originate in the financial system. For example, the adverse impacts of deteriorating financial conditions on consumer spending and residential investment in 2008-2009 reduced investment through their impact on GDP. Finally, unlike accelerator impacts on investment, changes in q represent exogenous shocks to the economy and typically have their greatest impact on investment earlier in a recession than does the accelerator.

8.3 The Roles of Cash Flow and Credit Constraints

I find that cash flow has an insignificant impact on investment when added to my empirical model. As a test of the role of cash flow, I added

$$c5\ \sigma_{mt} \left(\frac{1}{j} \sum_{i=1}^{j} CF_{t-i} - \widehat{CF} \right)$$

to each investment equation, where $c5$ is an estimated coefficient, CF is the ratio of nonfinancial corporate cash flow to potential GDP, and $\widehat{CF}$ is its sample average. (I subtracted

the sample average so as not to distort estimates of α^*.) For j equal to 1 or 4, the estimate of $c5$ is negative. For $j = 8$, $c5$ is statistically insignificant and small: each additional dollar of cash flow ultimately results in less than 6 cents of additional investment. I conclude that cash flow's significance in traditional q models results from its role as a proxy for a missing accelerator effect rather than as a measure of liquidity constraints.

While cash flow appears to have little impact on aggregate investment, it may nonetheless be true that financial crises hinder investment beyond the impacts captured by q, especially for inventories. The model developed in this paper assumes that businesses can obtain whatever financing they want at the going rate. That may not be true in financial crises, such as the one in 2008-2009. As a percent of the net stock of capital, the largest negative error for overall private business investment in the sample period occurred in the second quarter of 2009. Nearly all of the shortfall was in inventories, probably the easiest component of investment to adjust quickly in response to credit constraints.

8.4 Implications for the Sources of Capital Income

The first step in determining the sources of capital income is to obtain an estimate of $1/\eta$, the inverse of the elasticity of demand. From equation (26),

$$1/\eta_t = 1 - \frac{w_t H_t N_t}{p_t Y_t} - \alpha_t^* S_t x_t.$$

I calculate labor income, $w_t H_t N_t$, as labor compensation plus 65 percent of proprietors' income.

Using that estimate of $1/\eta$, I can calculate capital's coefficient of production α in the sectors using nonfarm BFI. Rearranging the definition of α_t^*, I can solve for α_t as

$$\alpha_t = \alpha_t^* S_t / \left(1 - 1/\eta_t\right).$$

Figure 8 shows the Kalman filter estimate of α_t^* and the resulting estimate of α_t. The estimate of α_t is larger than that of α_t^* for two reasons. First, α_t includes mining-specific capital, while α_t^* does not. Second, a positive $1/\eta_t$ reduces the net revenue from additional investment, reducing α_t^* for a given value of α_t.

Armed with the estimate of $1/\eta_t$, I can decompose capital's share of income into the share going to measured capital and the share going to unmeasured intangible capital. Capital's share of income is one minus labor's share, or $1 - \frac{w_t H_t N_t}{p_t Y_t}$. Unmeasured intangible capital receives share $1/\eta_t$ of total income. The remainder of capital income is the income of fixed capital, inventories, and IPP.

Figure 9 shows the resulting decomposition of capital's share of income, with measured

capital's share of income calculated by subtracting an H-P filtered estimate of $1/\eta_t$ from total capital's share of income. The increase in capital's share of income in the 1980s stemmed from a rise in measured capital's share of income, corresponding to a higher value of α_t. However, the increase in capital's share of income since about 2000 is the result of a sharp increase in the income generated by unmeasured intangible capital. Unless the share of income going to such capital reverts back to its historical share, capital's share of income will remain high.

That increasing share of income going to unmeasured intangible capital may have important effects on aggregate demand, interest rates, and inflation. The model developed in this paper implies that such income does not generate measured investment, which depends only on the market value of measured capital. In addition, such income only generates consumer spending through the wealth effect and through dividend and interest income, since it does not go to workers. Unless the wealth effect is large or the income of UIC generates demand in some other way, its increasing share of total income reduces aggregate demand. Lower real interest rates than in the past may then be needed to restore equilibrium to the economy. An increase in profit margins, due to capital's higher share of income, could help to explain why price inflation did not slow further when unemployment was high during and after the financial crisis of 2008-2009.

8.5 Implications for Market Value and Tobin's q

The model developed in this paper can also be used to determine how much of the total market value of firms to attribute to their various assets. To do that for nonfinancial corporate business, I solved for the rpy_t that satisfy

$$V_t = \sum_i V_{it} - c_V p_t \bar{Y}_t$$

using notation from equation (30).

Figure 10 shows the market value of all nonfinancial assets and the market value of nonfinancial assets excluding UIC, both relative to potential GDP. The two measures move roughly in parallel until the early 2000s, when the rise in UIC's share of income causes them to diverge. Estimated using the assumptions discussed above, UIC accounted for about a third of the market value of all nonfinancial assets of nonfinancial corporate business in 2013.

Those estimates of market value also allow us to construct an alternative measure of Tobin's q that uses only the market value and replacement cost of nonresidential fixed capital (equipment, structures, and IPP). Figure 11 compares that measure of Tobin's q with the traditional measure. The two measures are quite close until the early 2000s. After that, the inclusion of the market value of unmeasured intangible capital in the numerator of the

traditional measure of Tobin's q causes the two measures to diverge. According to that traditional measure, the ratio of the market value of capital to its replacement cost at the end of 2013 was the highest it had ever been except for a brief period at the peak of the high-tech bubble. Adjusted to exclude the market value of UIC, however, Tobin's q was no higher at the end of 2013 than it had been during much of the 2000s, prior to the financial crisis. That latter view helps explain why business investment remained relatively weak at the end of 2013.

8.6 Sensitivity Analysis

Thus far, I have assumed values for some key variables, including the service life of unmeasured intangible capital (δ_I), the sample-average for the real rate of return less productivity growth ($\widehat{rpy}$), and the standard error of α^* in the Kalman filter. For the most part, changing those values has little impact on the parameter estimates in the investment equations but changes the estimate of income earned by unmeasured intangible capital.

When the assumed value of δ_I is reduced from 15 years to 10 years, the estimated parameters are little changed (see Table 2). The lower value for δ_I reduces the market value of UIC and thus $d\hat{V}_{It}/d\widehat{rpy}$ for UIC. That increases the $d\hat{V}_{mt}/d\widehat{rpy}$ for measured capital, reducing the estimate of $c4$ from 0.20 to 0.19.

The results are qualitatively similar when the assumed value of $\widehat{rpy}$ is increased from 4.0 percent to 4.5 percent. Once again, the estimated parameters are little changed. One can see from equation (29) that the higher $\widehat{rpy}$ requires an increase in α^*—of 0.015 to 0.018—to generate the same level of investment. That higher estimate of α^* shifts between 1.2 percent and 1.5 percent of gross domestic income from UIC to measured capital. An increased $\widehat{rpy}$ also reduces the $\hat{V}_{mt}$, changing the required c_V from -0.23 to -0.19.

Increasing the standard error of α^* in the Kalman filter from 0.0005 to 0.001 improves the overall fit at the expense of an implausible swing in estimated income from UIC. The increase in the standard error allows α^* to capture more of the movements in investment, boosting the log likelihood of the system of equations from 4340 to 4426. The general pattern of the estimated β and γ is similar to that of the primary specification, but the estimates of $c2$ and $c3$ fall considerably and estimated $c4$ increases slightly. The improved fit of the investment equations is accomplished by an increase in α between 1972 and 1985 that is much larger than the simultaneous increase in overall capital's share of income, implying an implausible temporary drop in UIC's share of income.

9 Conclusions

The model developed in this paper features two basic innovations. First, capital is modeled as complementary putty-clay, solving empirical problems associated with putty-putty capital and gaps between theory and empirics in traditional putty-clay models. Second, I use investment to identify the sources of capital income and the market value of capital.

Those innovations have important implications for how investment behaves and how it should be modeled. Empirically, fluctuations in output are the key driver of investment. Although Tobin's q is statistically very significant, it plays a less important role in driving investment. The model implies that a DSGE model in which investment is calibrated to respond symmetrically to fluctuations in output and the cost of capital, as in the neoclassical model, will get at least one of those responses wrong.

The implications for the sources of capital income and wealth are perhaps even more important. I find that the increase in capital's share of total GDP since about 2000 is due to an increase in income accruing to unmeasured intangible capital (UIC). If permanent, that shift will have important effects on the economy. Because such income likely has a smaller impact on demand than income accruing to labor or measured capital, the economy may require lower real interest rates to maintain full employment. To the extent that such income is more concentrated than labor income, it also increases income inequality.

Increased income going to unmeasured intangible capital could have positive effects as well. To the extent that firms can invest in UIC and that such investment responds to its market value in the same way that investment in measured capital does, greater income for UIC could increase firms' incentive to innovate, boosting the overall standard of living in the long run. The ways in which firms invest in UIC could be interesting topics.

Appendix

A The Effect of Productivity Growth on Capital's Share of Income

To determine the effect of productivity growth on capital's share of income, I examine a simple case in which there is no time to build, so that the number of units of capital (M) equals the number of workers (N). The value of cash flow at time t discounted to time t_0 is

$$DCF_t = \{(1 - ub_t)\left[p_t Y_t - w_t H_t N_t\right] - p\tau_t k_t F_t\} \exp\left(-\int_{i=t_0}^{t} r_i di\right).$$

To solve that as an Euler equation, I express DCF_t as a function of state variables $\bar{k}_t$ and N_t by replacing k_t with $\bar{k}_t \exp\left(\frac{d\bar{k}_t/dt}{\bar{k}_t}\frac{N_t}{dN_t/dt+\delta N_t}\right)$ (by inverting equation (6)) and F_t with $\frac{dN_t}{dt} + \delta N_t$. The Euler equation for the choice of N_t is

$$\frac{\partial DCF_t}{\partial N_t} - \frac{d}{dt}\frac{\partial DCF_t}{\partial\left(dN_t/dt\right)} = 0.$$

To evaluate that equation, I assume static expectations for $k_t/\bar{k}_t$, for tax law, and for the determinants of resale value. In that case, $d\log\left(\frac{k_t}{\bar{k}_t}\right)/dt = 0$ and $\frac{d(p\tau_t k_t)}{dt} = p\tau_t k_t\left(\dot{p} + \dot{y}\right)$, and the solution to the Euler equation is

$$(1 - ub_t)\left(\left[1 - \frac{1}{\eta_t}\right]\frac{p_t Y_t}{N_t} - w_t H_t\right) - p\tau_t k_t \frac{F_t}{N_t}\log\left(\frac{k_t}{\bar{k}_t}\right)$$
$$- \left(\delta + r_t - \dot{p}_t - \dot{y}_t\right) p\tau_t k_t\left[1 - \log\left(\frac{k_t}{\bar{k}_t}\right)\right] = 0.$$

In order to convert that equation into an expression for capital's share of income, substitute for $p\tau_t k_t$ using equation (11) and for $\frac{F_t}{N_t}$ with $\delta + \frac{dN_t/N_t}{dt}$. After further rearranging, that yields

$$\frac{\left(1 - 1/\eta_t\right) p_t Y_t - w_t H_t N_t}{p_t Y_t} = \left(1 - \frac{1}{\eta_t}\right)\alpha_t x_t, \tag{32}$$

where $x_t \equiv 1 + \log\left(\frac{k_t}{\bar{k}_t}\right)\frac{\delta + \frac{dN_t/N_t}{dt}}{\delta + r_t - \dot{p}_t - \dot{y}_t}$.

Because $\frac{1}{\eta_t} p_t Y_t$ is the income of unmeasurable capital, the left-hand side of equation (32) is the share of income earned by measurable capital.

If productivity grows over time, new capital is of higher quality than existing capital, so $\log\left(k_t/\bar{k}_t\right)$ is positive. (New capital will also be of higher quality than existing capital if the real price of capital falls over time.) The growth rate of the number of workers $\frac{dN_t/N_t}{dt}$

has typically been smaller than the real rate of return less productivity growth $r_t - \dot{p}_t - \dot{y}_t$. Consequently, $x_t < 1$ and the right-hand side of equation (32) is less than $(1 - 1/\eta_t)\,\alpha_t$. In a world of perfect competition ($\eta_t = \infty$), capital's share of income is less than its coefficient in production (α_t).

When there are many types of capital, x_t is a weighted average over all types of capital using the k_t, $\bar{k}_t$, and δ specific to each type of capital:

$$x_t = 1 + \sum_m \frac{M_{mt}}{M_t} \log\left(\frac{k_{mt}}{\bar{k}_{mt}}\right) \frac{\delta_m + \frac{dM_{mt}/M_{mt}}{dt}}{\delta_m + r_t - \dot{p}_t - \dot{y}_t}.$$

In the empirical work, I estimate that expression using actual values of the $\frac{M_{mt}}{M_t}$ combined with a steady-state growth path estimate of $\frac{k_{mt}}{\bar{k}_{mt}}$, the sample-average of $r_t - \dot{p}_t - \dot{y}_t$, and sample-average $\frac{dM_t/M_t}{dt}$ for $\frac{dM_{mt}/M_{mt}}{dt}$. Using actual values for those variables would make x_t unrealistically volatile over time.

B New Units of Capital

B.1 One Type of Capital

This section completes the derivation of new units of capital from the main text. Rearranging equation (18), dividing through by ${}_{t'}\bar{y}_{t+T/2}$, and substituting $\bar{N}$ for $\bar{Y}/\bar{y}$ yields

$$\frac{{}_{t'}Y_{t+T/2}}{{}_{t'}\bar{y}_{t+T/2}} = \psi\ {}_{t'}\bar{N}_{t+T/2} \frac{Y_{t'}/\bar{y}_{t'}}{\bar{N}_{t'}} - (1-\psi)_{t'}\,\bar{N}_{t+T/2}.$$

The firm assumes that $\bar{N}$ will grow at rate $\bar{n}_{t'}$ between t' and $t + T/2$, so

$${}_{t'}\bar{N}_{t+T/2} = \bar{N}_{t'}\left(1 + \bar{n}_{t'}\right)^{t+T/2-t'}.$$

Combining those two equations, I find that

$$\frac{{}_{t'}Y_{t+T/2}}{{}_{t'}\bar{y}_{t+T/2}} = \psi \frac{Y_{t'}}{\bar{y}_{t'}}\left(1 + \bar{n}_{t'}\right)^{t+T/2-t'} + (1-\psi)\,\bar{N}_{t'}\left(1 + \bar{n}_{t'}\right)^{t+T/2-t'}.$$

Thus, the expected future ratio of output to output per hour depends on the ratio of output to cyclically adjusted output per worker and full employment at the time orders are made.

Firms' orders at time t' equal growth in the desired number of units of capital since those firms' last order, or

$$\frac{{}_{t'}Y_{t+T/2}}{{}_{t'}\bar{y}_{t+T/2}} - \frac{{}_{t'-T}Y_{t-T/2}}{{}_{t'-T}\bar{y}_{t-T/2}},$$

plus replacement demand. Multiplying by the fraction of firms ordering in each period ($1/T$),

orders (O) are

$$\begin{aligned} O_{t'} &= \psi\, d\left[\frac{Y_{t'}}{\bar{y}_{t'}}\left(1+\bar{n}_{t'}\right)^{t+T/2-t'}, T\right]/T \\ &\quad + (1-\psi)\, d\left[\bar{N}_{t'}\left(1+\bar{n}_{t'}\right)^{t+T/2-t'}, T\right]/T + \frac{\left[1-(1-\delta)^T\right] M_{t-T/2}}{T}, \end{aligned}$$

where, to simplify notation, $d\left[X_t, i\right]$ replaces $X_t - X_{t-i}$. Deliveries of new units of capital at time t (F_t) are the average of orders placed in periods t' through $t-1$:

$$\begin{aligned} F_t &= \psi \sum_{i=1}^{t-t'} \frac{d\left[\left(Y_{t-i}/\bar{y}_{t-i}\right)\left(1+\bar{n}_{t-i}\right)^{t-t'+T/2}, T\right]}{(t-t')\times T} \\ &\quad + (1-\psi)\sum_{i=1}^{t-t'} \frac{d\left[\bar{N}_{t-i}\left(1+\bar{n}_{t-i}\right)^{t-t'+T/2}, T\right]}{(t-t')\times T} + \sum_{i=1}^{t-t'} \frac{\left[1-(1-\delta)^T\right] M_{t-T/2-i}}{(t-t')\times T}. \end{aligned}$$

To simplify the equation below and the empirical work, I replace the expression for depreciation with δM_{t-1}, which is units depreciating in period t.

Now consider a world in which investment cycles can vary in length, so that instead of a single T there are many T_j. Because the degree of expected convergence of employment toward full employment depends on the length of the investment cycle, a unique ψ_j corresponds to each T_j. Then the number of new units of capital put in place becomes

$$\begin{aligned} F_t &= \sum_{i=1}^{t-t'}\sum_j \beta_j \frac{d\left[\left(Y_{t-i}/\bar{y}_{t-i}\right)\left(1+\bar{n}_{t-i}\right)^{t-t'+T_j/2}, T_j\right]}{(t-t')\times T_j} \\ &\quad + \sum_{i=1}^{t-t'}\sum_j \gamma_j \frac{d\left[\bar{N}_{t-i}\left(1+\bar{n}_{t-i}\right)^{t-t'+T_j/2}, T_j\right]}{(t-t')\times T_j} + \delta M_{t-1}, \end{aligned} \tag{33}$$

where $\beta_j = \phi_j \psi_j$, $\gamma_j = \phi_j\left(1-\psi_j\right)$, and $\sum_j \beta_j + \sum_j \gamma_j = 1$.

B.2 Many Types of Capital

Now consider a world in which there are many types of capital, e.g., computers, software, aircraft, and office buildings. I assume the same delivery lag ($t - t'_m$) applies to all type-m capital, but that the length of the investment cycle (T_{mj}) and the convergence parameter (ψ_{mj}) differ across types of capital as well as establishments. Thus, the pair (T_{mj}, ψ_{mj}) apply to fraction ϕ_{mj} of investment in type-m capital.

To distribute investment among the different types of capital, I assume that, absent cyclical effects, type-m capital would account for a share σ_{mt} of the nonfarm non-mining units delivered at time t. I implement that assumption by multiplying each term of equation

(33) by σ_{mt}. The number of new units of type-m capital delivered at time t is

$$\begin{aligned} F_{mt} &= \sigma_{mt} \sum_{i=1}^{t-t'_m} \sum_j \beta_{mj} \frac{d\left[\left(Y_{t-i}/\tilde{y}_{t-i}\right)\left(1+\bar{n}_{t-i}\right)^{t-t'_m+T_{mj}/2}, T_{mj}\right]}{\left(t-t'_m\right)\times T_{mj}} \\ &\quad +\sigma_{mt} \sum_{i=1}^{t-t'_m} \sum_j \gamma_{mj} \frac{d\left[\bar{N}_{t-i}\left(1+\bar{n}_{t-i}\right)^{t-t'_m+T_{mj}/2}, T_{mj}\right]}{\left(t-t'_m\right)\times T_{mj}} + \sigma_{mt} \sum_{m\notin m\&f} \delta_m M_{m,t-1}, \end{aligned}$$

where $m \notin m\&f$ denotes nonfarm non-mining types of capital.

C Proof That $\lambda_t K_t$ Is the Market Value of Capital

Proposition 1 *Suppose the firm is a price-maker in the output market. If the production function is homogeneous in employment, N, and aggregate capital, K, then*

$$\lambda_t K_t = V_t - D_t - V_t^{UF} - V_t^{UE} - V_t^{W} - V_t^{B}. \tag{34}$$

Proof. The first order condition $\frac{\partial\left(\lambda_t \exp\left[-\int_{i=0}^{t} r_i di\right]\right)}{\partial t} = -\frac{\partial H_t}{\partial K_t}$ yields

$$\begin{aligned} \frac{\partial \lambda_t}{\partial t} - \lambda_t r_t &= -\left(1-ub_t\right)\frac{d\left(p_t Y_t\right)}{dK_t} + \lambda_t K_t \frac{F_t}{M_t}\frac{1}{K_t} \\ &\quad -\lambda_t\left[\frac{F_t}{M_t}\log\left(\frac{k_t M_t}{K_t}\right) + \frac{dM_t/M_t}{dt}\right] \end{aligned} \tag{35}$$

after dividing through by $\exp\left[-\int_{i=0}^{t} r_i di\right]$. Because demand is inelastic,

$$\frac{d\left(p_t Y_t\right)}{dK_t} = \left(1-\frac{1}{\eta_t}\right) p_t \frac{\partial Y_t}{\partial K_t} K_t. \tag{36}$$

Multiplying equation (35) through by K_t, substituting for $\frac{d(p_t Y_t)}{dK_t}$ using (36), for $\lambda_t K_t$ using equation (20) and for the last line of equation (35) using equation (6), and combining terms, I obtain

$$\frac{\partial\left(\lambda_t K_t\right)}{\partial t} - r_t\left(\lambda_t K_t\right) = -\left(1-ub_t\right)\left(1-\frac{1}{\eta_t}\right) p_t \frac{\partial Y_t}{\partial K_t} K_t + p\tau_t k_t F_t.$$

The solution to that differential equation is

$$\lambda_t K_t = \int_{j=t}^{\infty} \left\{ \begin{array}{c} \left(1-ub_j\right)\left(1-\frac{1}{\eta_j}\right) p_j \frac{\partial Y_j}{\partial K_j} K_j \\ -p\tau_j k_j F_j \end{array} \right\} \exp\left(-\int_{i=t}^{j} r_i di\right) dj. \tag{37}$$

I assume that unmeasurable capital is what causes firms' price elasticity of demand to differ from infinity, so the after-tax income from unmeasurable capital is $\left(1-ub_t\right)\frac{1}{\eta_t} p_t Y_t$.

Let V^{UE} be the market value of existing unmeasurable capital and V^{UF} be the PDV of the after-tax profits due to unmeasurable capital yet to be created, then

$$V_t^{UE} + V_t^{UF} = \int_{j=t}^{\infty} (1 - ub_j) \frac{1}{\eta_j} p_j Y_j \exp\left(-\int_{i=t}^{j} r_i di\right) dj.$$

The market value of the firm is the PDV of future cash flow (V_t) less V_t^{UF}.

To evaluate the remainder of V_t, I use Euler equations. First, consider the PDV of the portion of V_t corresponding to time j, the discounted cash flow

$$DCF_j = \left\{ \begin{array}{c} (1 - ub_j) \left[\begin{array}{c} p_j Y_j - \frac{1}{1-\alpha_j x_j} w_j H_j N_j \\ + \frac{\alpha_j}{1-\alpha_j x_j} w_j \bar{H}_j M_j - C_j \end{array} \right] \\ + u_j i_j^B B_j - p\tau_j k_j F_j \end{array} \right\} \exp\left(-\int_{i=t}^{j} r_i di\right),$$

as a function of the state variable N_j. Then the Euler equation is

$$\frac{\partial DCF_j}{\partial N_j} = -\frac{d}{dj} \frac{\partial DCF_j}{\partial (dN_j/dj)}.$$

To evaluate the right-hand side of that equation, I use the simplifying assumptions that $w_j H_j N_j$ grows at rate $\dot{p}_t + \dot{y}_t$ and that $d\left(\frac{dN_t/N_t}{dt} - \bar{n}_t\right)/dj = 0$. Some algebra yields

$$\begin{aligned} &\left(1 - 1/\eta_j\right) \frac{p_j Y_j}{N_j} - \frac{1}{1 - \alpha_j x_j} w_j H_j N_j - \frac{1}{2} c_N w_j \left(\frac{dN_j/N_j}{dj} - \bar{n}_j\right)^2 H_j N_j \\ &- \frac{1}{2} c_H w_j \left(\frac{H_j}{\bar{H}_j} - 1\right)^2 \bar{H}_j N_j - c_M w_j H_j \left(\frac{N_j}{M_j} - 1\right) N_j \\ &\qquad = r p y_j c_N w_j \left(\frac{dN_j/N_j}{dj} - n_j\right) H_j N_j. \end{aligned}$$

Now consider DCF_j as a function of the state variable M_j. To evaluate DCF_j I make the substitutions

$$k_j = \bar{k}_j \exp\left(\frac{d\bar{k}_j/dj}{\bar{k}_j} \frac{M_j}{dM_j/dj + \delta M_j}\right) \text{ and}$$

$$F_j = \frac{dM_j}{dj} + \delta M_j.$$

The Euler equation is

$$\frac{\partial DCF_j}{\partial M_j} = -\frac{d}{dj} \frac{\partial DCF_j}{\partial (dM_j/dj)}.$$

To evaluate the right-hand side of that equation, I use the simplifying assumptions that $k_j/\bar{k}_j$ is constant and that $p\tau_j k_j$ grows at rate $\dot{p}_t + \dot{y}_t$. After some algebra, and using equation (11)

to substitute for $p\tau_j k_j$, I have

$$\frac{\alpha_j}{1-\alpha_j x_j} w_j \bar{H}_j M_j - \frac{1}{2} c_M w_j H_j \left(\frac{N_j}{M_j} - 1\right)^2 M_j + c_M w_j H_j \left(\frac{N_j}{M_j} - 1\right) N_j =$$
$$\alpha_j \left(1 - 1/\eta_j\right) p_j Y_j + \alpha_j \left(1 - 1/\eta_j\right) p_j Y_j \log\left(\frac{k_j}{\bar{k}_j}\right) \frac{\frac{dM_j/dj}{M_j} - rpy_j}{\delta + rpy_j}.$$

Summing that equation with the result from the first Euler equation yields

$$(1-\alpha_j)\left(1 - 1/\eta_j\right) p_j Y_j - \frac{1}{1-\alpha_j x_j} w_j H_j N_j + \frac{\alpha_j}{1-\alpha_j x_j} w_j \bar{H}_j M_j - C_j =$$
$$rpy_j c_N w_j \left(\frac{dN_j/N_j}{dj} - \bar{n}_j\right) H_j N_j + \alpha_j \left(1 - 1/\eta_j\right) p_j Y_j \log\left(\frac{k_j}{\bar{k}_j}\right) \frac{\frac{dM_j/dj}{M_j} - rpy_j}{\delta + rpy_j}.$$

To evaluate V_t using that equation, note that I can substitute for the first term on the left-hand using

$$\frac{\partial Y_j}{\partial N_j} N_j = (1-\alpha_j) p_j Y_j$$

when the derivative $\partial Y_j / \partial N_j$ of equation (7) is taken with K_j held constant. Multiplying through by $(1 - ub_j)$ and integrating both sides over j, I have

$$\int_{j=t}^{\infty} (1 - ub_j) \left\{ \begin{array}{c} \left(1 - \frac{1}{\eta_j}\right) p_j \frac{\partial Y_j}{\partial N_j} N_j \\ -\frac{1}{1-\alpha_j x_j} w_j H_j N_j \\ +\frac{\alpha_j}{1-\alpha_j x_j} w_j \bar{H}_j M_j - C_j \end{array} \right\} \exp\left(-\int_{i=t}^{j} r_i di\right) dj = V_t^W, \tag{38}$$

where

$$V_t^W = \int_{j=t}^{\infty} (1 - ub_j) \left\{ \begin{array}{c} rpy_j c_N w_j \left(\frac{dN_j/N_j}{dj} - \bar{n}_j\right) H_j N_j \\ +\alpha_j \left(1 - 1/\eta_j\right) p_j Y_j \log\left(\frac{k_j}{\bar{k}_j}\right) \frac{\frac{dM_j/dj}{M_j} - rpy_j}{\delta + rpy_j} \end{array} \right\} \exp\left(-\int_{i=t}^{j} r_i di\right) dj.$$

The portion of V_t^W dependent on $\left(\frac{dN_j/N_j}{dt} - \bar{n}_j\right)$ is very small, since $\bar{n}_j$ is just growth of smoothed N_j. Thus, V_t^W consists primarily of the reduction in capital's share of income below that implied by α_j because of growing productivity.

Because the production function is homogeneous of degree one in N_j and K_j,

$$Y_j = \frac{\partial Y_j}{\partial N_j} N_j + \frac{\partial Y_j}{\partial K_j} K_j.$$

Summing together equations (38) and (37) and making that substitution yields

$$\lambda_t K_t + V_t^W = \int_{j=t}^{\infty} \left\{ (1-ub_j) \begin{bmatrix} \left(1-\frac{1}{\eta_j}\right) p_j Y_j \\ -\frac{1}{1-\alpha_j x_j} w_j H_j N_j \\ +\frac{\alpha_j}{1-\alpha_j x_j} w_j \bar{H}_j M_j - C_j \end{bmatrix} \begin{matrix} \\ \\ -p\tau_j k_j F_j \end{matrix} \right\} \exp\left(-\int_{i=t}^{j} r_i di\right) dj.$$

The right-hand side of that equation is $V_t - D_t - V_t^{UF} - V_t^{UE} - V_t^B$, so I have proved equation (34).

The proof generalizes to the case in which there are many types of capital. Other than summing the differential equation (37) over all types of capital, the only difference with the case of one type of capital is that the λ_{mt} for land and inventories include their resale value.

D Notes on Data

D.1 Estimated Variables Used in the Investment Equations

To estimate $\bar{N}_t$, I assume that the ratio $N_t/\bar{N}_t$ is negatively related to the difference between the actual unemployment rate (ru_t) and the underlying long-term rate of unemployment ($\overline{ru}_t$):

$$\log\left(\frac{N_t}{\bar{N}_t}\right) = -\zeta\left(ru_t - \overline{ru}_t\right). \tag{39}$$

Data for $\overline{ru}_t$ are from the Congressional Budget Office (2014b). I assume that $\log\left(\bar{N}_t\right)$ follows a random walk with drift

$$\Delta \log\left(\bar{N}_t\right) = \varphi + \epsilon_t,$$

where Δ denotes a first difference. I estimate ζ using

$$\Delta \log\left(N_t\right) = \varphi - \zeta\ \Delta\left(ru_t - \overline{ru}_t\right) + \epsilon_t$$

and use that estimate of ζ to calculate $\bar{N}_t$ by inverting equation (39).

The variable $\bar{H}_t$ is calculated by taking an H-P filter of H_t. I can then use equation (12) to solve for $\bar{y}_t$.

D.2 Market Value of Debt

To convert the par value of interest-bearing liabilities into market value, I first regress the ratio of interest paid to the par value of those liabilities on past interest rates. That

determines the interest rate and the average maturity, denoted T_M. Rather than making a separate calculation for each maturity, I assume that all the liabilities held at time t were issued at time $t - T_M$ at the average interest rate over the period $t - 2T_M$ to t. Let that average rate be r_0 and the actual interest rate at time t be r_1. Then the market value is the par value times

$$\frac{r_0}{r_1}\left(1 - \frac{1}{(1+r_1)^{T_M}}\right) + \frac{1}{(1+r_1)^{T_M}}.$$

The market value of assets is calculated using an analagous procedure.

D.3 Tax Treatment of New Capital

To calculate the tax treatment of capital, I assume all capital is subject to the corporate income tax. Data for the methods of depreciation taken (straight-line, accelerated, sum of digits, and expensing), tax lifetimes, declining-balance parameters, rates of investment tax credit, and the basis adjustment for the investment credit are taken from Gravelle (1994), updated for provisions allowing bonus depreciation since 2002. The take-up rate for expensing and partial expensing is assumed less than 1, based on Knittel (2007).

The effective tax rate on new capital can differ from the average effective tax rate for several reasons. First, the effective tax rate for nonprofit institutions is zero. Second, the effective tax rate for S corporations is the personal tax rate rather than the corporate rate. Third, loss-making firms may not be able to take advantage of depreciation allowances or may have to take them in later years. Finally, not all income is subject to tax, even for profitable firms. Based on those considerations, I multiply the statutory federal rate by a factor g, which is 0.80 for all firms investing in capital and 0.85 for nonfinancial corporations. The corporate tax rate is then

$$u_t = g \times uf_t + usl_t * (1 - g \times uf_t),$$

where uf_t is the federal statutory corporate income tax rate, from Gravelle, and usl_t is the average state and local corporate tax rate, equal to state and local corporate tax collections divided by corporate profits before tax for domestic industries. The latter is multiplied by $(1 - g \times uf_t)$ because state and local taxes are deductible from federal tax.

The present discounted value of depreciation allowances requires an estimate of the nominal rate of return r, so given an estimate of $r_t - \dot{p}_t - \dot{y}_t$, estimates of $\dot{p}_t$ and $\dot{y}_t$ are needed. From the fourth quarter of 1968 on, raw $\dot{p}_t$ equals the change in the GDP price index expected over the next year from the Philadelphia Federal Reserve Bank's survey of professional

forecasters, plus the average difference in growth rates between p_t and the GDP price index. Because expected inflation has to cover the entire tax life of an asset, not just the next year, the actual $\dot{p}_t$ used is an average of raw $\dot{p}_t$ and its sample average. Estimates of $\dot{p}_t$ before the fourth quarter of 1968 are based on a regression of $\dot{p}_t$ on current and lagged growth of p_t and on the gap between the unemployment rate and CBO's estimate of the underlying long-term rate of unemployment after 1968. Expected $\dot{y}$ is assumed to equal 1.6 percent, the coefficient of time in a regression of $\log(\tilde{y})$ on a constant and time.

D.4 Economic Depreciation

The rate of economic depreciation is the inverse of the service life. Service lives for most types of capital are from the Bureau of Economic Analysis (2003). The service life for inventory technology is assumed to be 12 years, a rough average of service lives for equipment. The service life for unmeasured intangible capital is assumed to be 15 years.

D.5 Starting Values for Units of Capital

The calculation of the V_{mt} requires estimates of the M_{mt}. Those can be calculated after the beginning of the sample period using perpetual inventory equations, but require estimates at the beginning of the sample to start the process. I do that by estimating the number of new units of each type of capital along a steady-state growth path consistent with BEA's estimates of the capital stocks at the beginning of the sample.

Along a steady-state growth path, the real stock of type-m capital, as measured by BEA, is proportional to the ratio of nominal output to the price of type-m capital:

$$K_{mt}^B = \int_{i=0}^{\infty} I_{mt} \exp\left(-\left[\delta_m^B + \dot{p}_t + \dot{Y}_t - \dot{p}_{mt}\right] i\right) di = \frac{k_{mt} F_{mt}}{\delta_m^B + \dot{p}_t - \dot{p}_{mt} + \dot{Y}_t},$$

where $\dot{p}_{mt}$ and $\dot{Y}_t$ are the growth rates of p_{mt} and Y_t, and δ_m^B is the depreciation rate used by BEA. Substituting for k_{mt} using equation (11) and for F_{mt} using

$$F_{mt} = M_{mt}\left(\delta_m^B + \frac{dN_t/N_t}{dt}\right)$$

and rearranging, I can express the M_{mt} as functions of α_t^*, rpy_t, and observable variables. (Because I am assuming a steady-state growth path, I use long moving averages for the tax terms and average growth rates over many years.) I set rpy_t equal to 0.05 (somewhat higher than in the sample period because the ratio of capital income to market value was high in the 1950s) and solve for the α_t^* needed to equate nonfarm non-mining M_t with N_t at the beginning of the sample. I can then solve for the individual M_{mt}.

D.6 Units of Land

To estimate the ratio of units of land to units of structures, first note that equation (25) implies that the PDV of the market value in production (the λK) of a unit of land newly placed in service and the associated units of structure units are the same because the depreciation rates are the same. Given that the two units share the other determinants of market value, their λK will remain the same over their service lives. With non-geometric depreciation, the resale value of land is roughly a fraction $\frac{1.5\delta_m}{1.5\delta_m + r_t - \dot{p}_{Lt}}$ of the total market value. Combining those facts with the Bureau of Labor Statistics' estimate that the market value of land is 29 percent as large as the market value of structures in the nonfarm business sector, the ratio of the number of units of land to the number of units of structures is

$$0.29 \frac{r_t - \dot{p}_{Lt}}{1.5\delta_m + r_t - \dot{p}_{Lt}}.$$

Given the paucity of data for land, I assume that ratio is constant over the sample period, and calculate it using sample averages of the variables and a weighted average of the δ_m for each type of structure. To capture the relative scarcity of land, I assume that the price of land grows at rate $\dot{p} + \dot{y}$, meaning $r_t - \dot{p}_{Lt}$ can be replaced by rpy_t.

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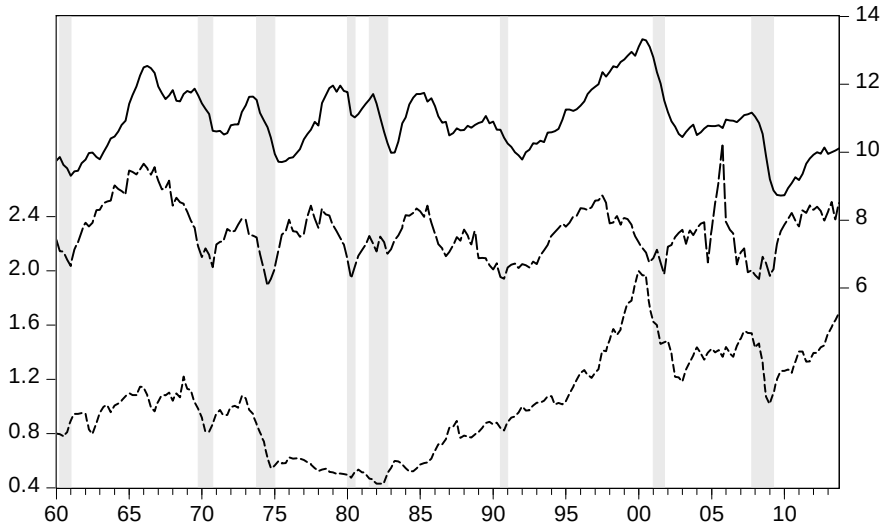
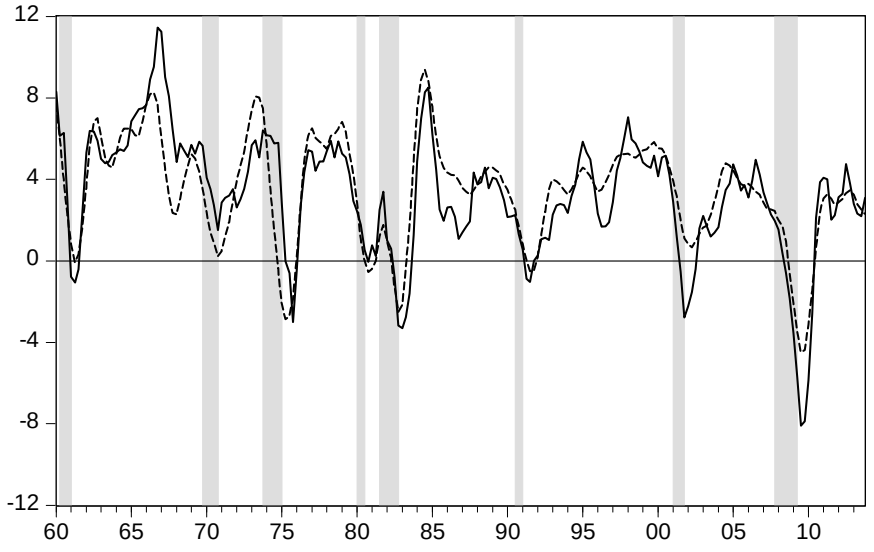
Figure 1. Investment, Cash Flow, and Tobin's q



——— Business Fixed Investment (percent of net stock of capital, right scale)
– – – – Nonfarm Nonfinancial Corporate Cash Flow (percent of net stock of capital, right scale)
- - - - - Tobin's q as Traditionally Measured (left scale)
Shading denotes recessions.

Figure 2. Inventories and Output
(Percent change from year earlier)



——— Real Nonfarm Inventories
– – – – Real Nonfarm Business GDP (five-quarter moving average)

Figure 3. Private Nonresidential Fixed Investment: Actual and Fitted
(Excluding mining and farming, percent of nominal net stock of capital)

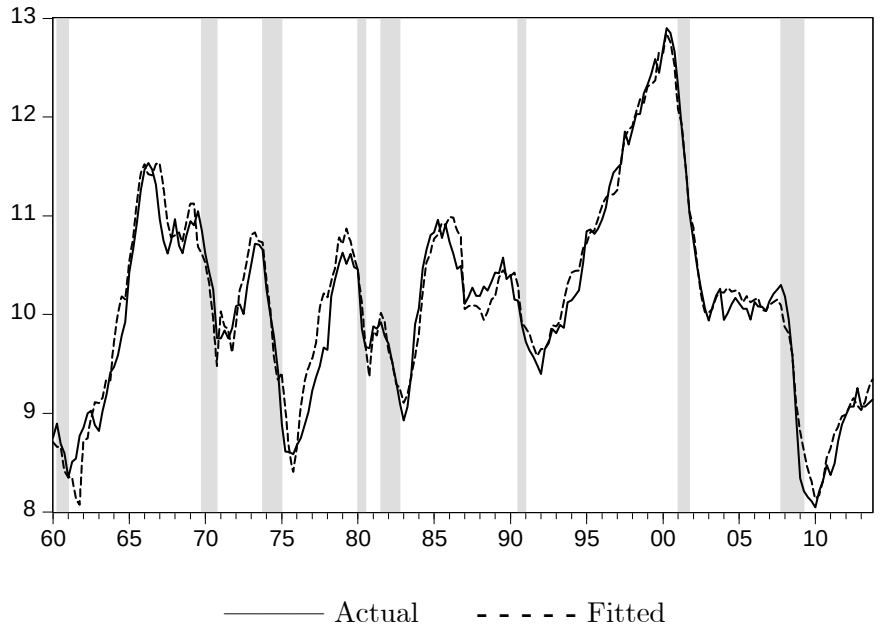


Figure 4. Nonfarm Inventory Investment: Actual and Fitted
(Percent of nominal net stock of capital)

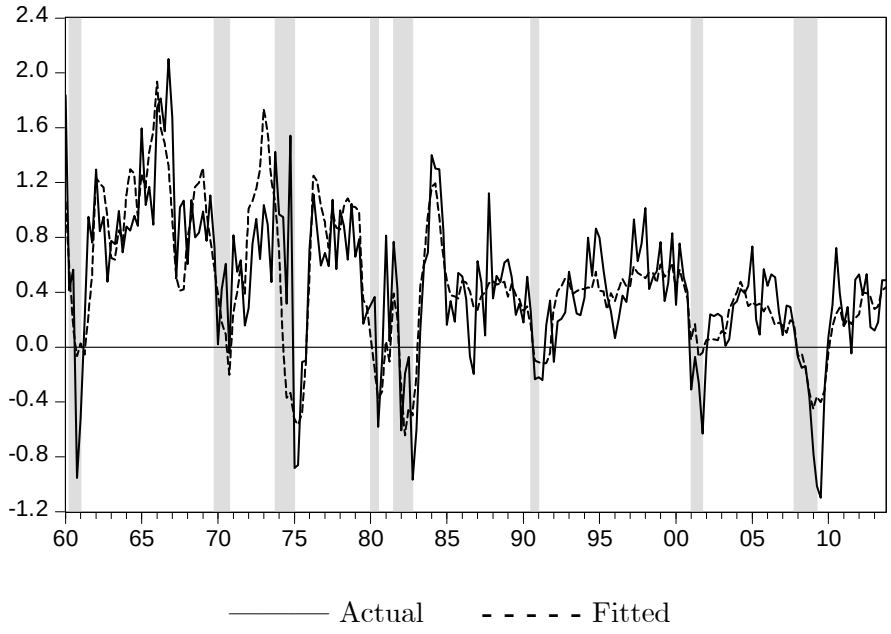


Figure 5. Private Nonresidential Fixed Investment: Actual and Fitted, Constant α^*
(Excluding mining and farming, percent of nominal net stock of capital)

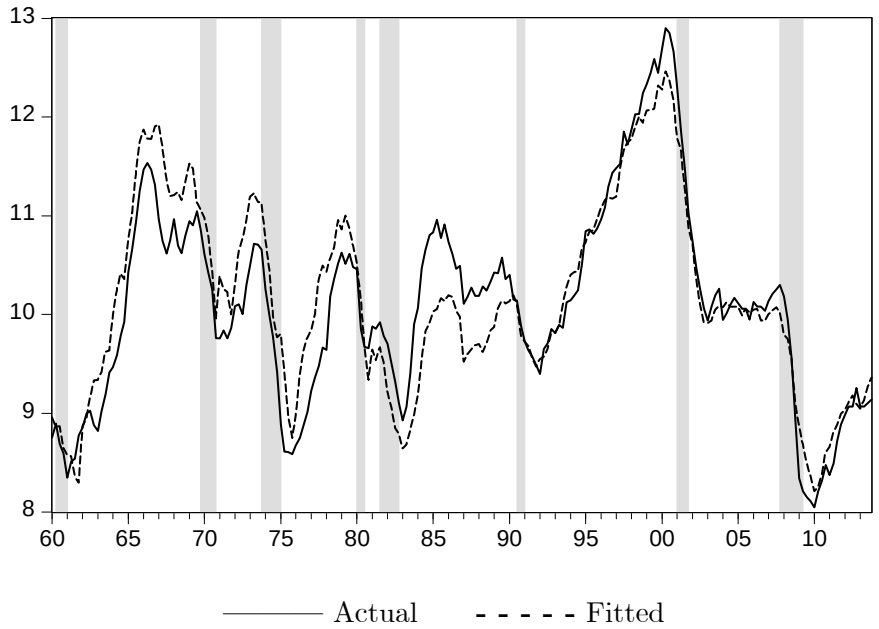


Figure 6. The Contribution of the Accelerator to the Growth of Investment
(Real nonresidential fixed investment, percent change from year earlier)

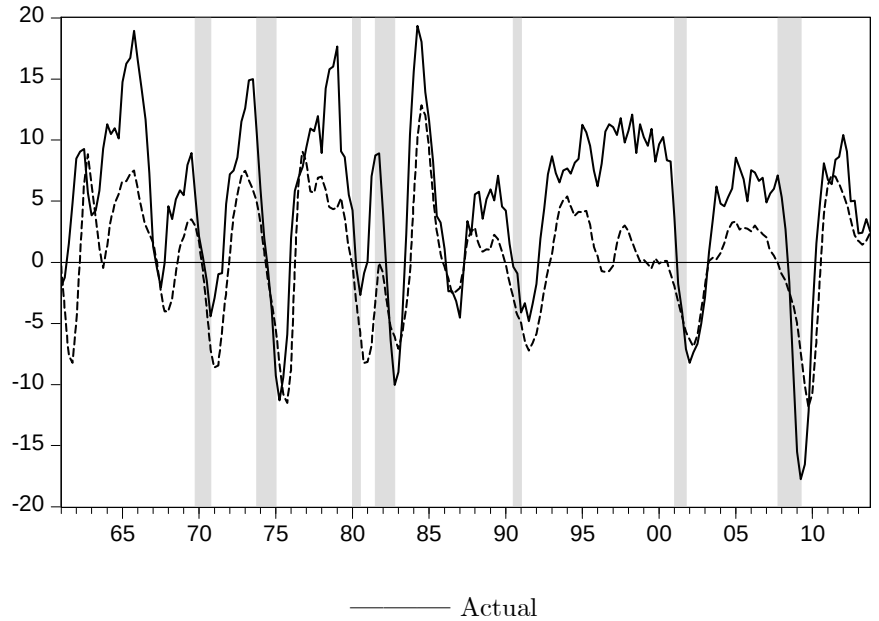


Figure 7. The Contribution of Tobin's q to the Growth of Investment
(Real nonresidential fixed investment, percent change from year earlier)

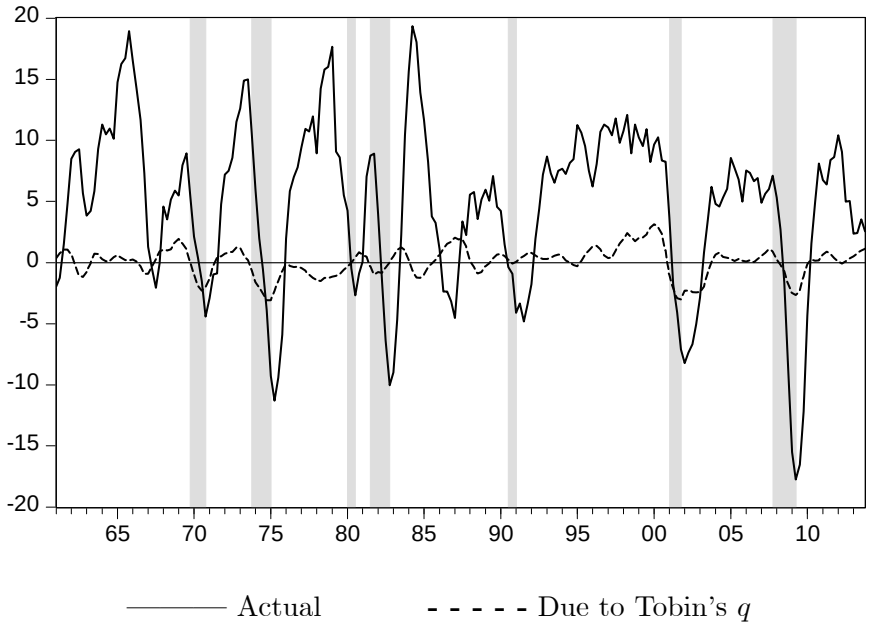


Figure 8. Estimates of α and α^* Using Variable α^*

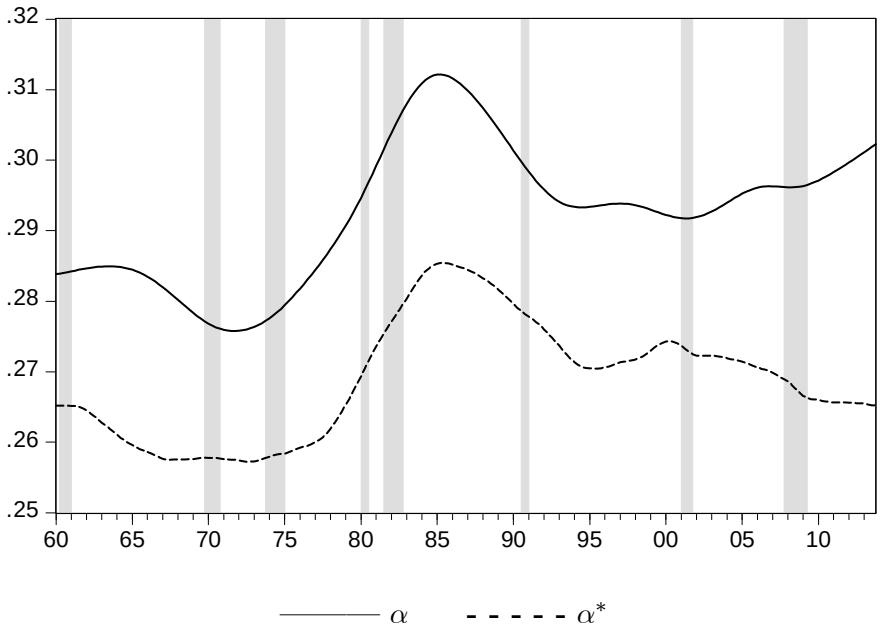


Figure 9. Capital's Share of Income
(Percent of output of sectors using private nonresidential fixed capital)

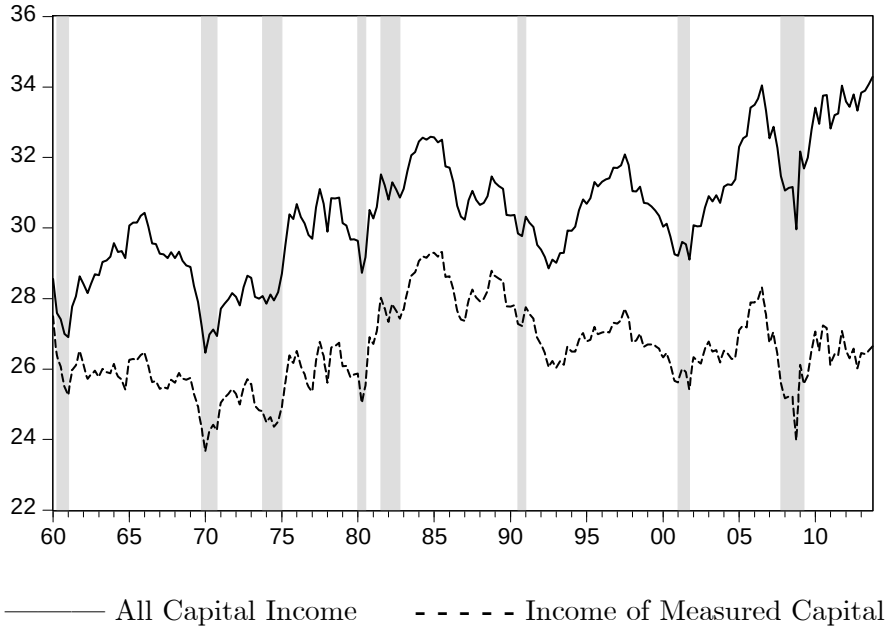


Figure 10. Market Value of Capital
(Nonfarm nonfinancial corporations, ratio to potential GDP)

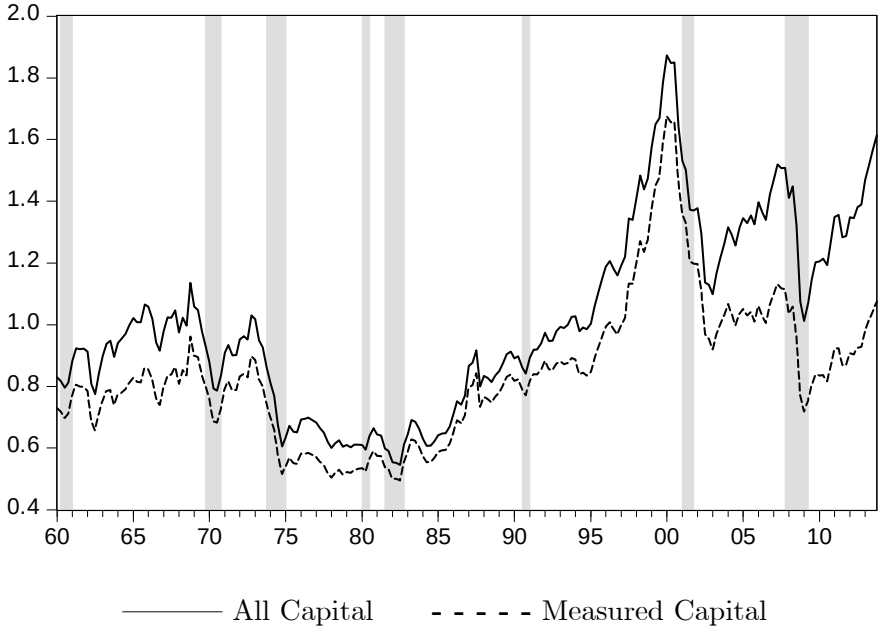


Figure 11. Tobin's q

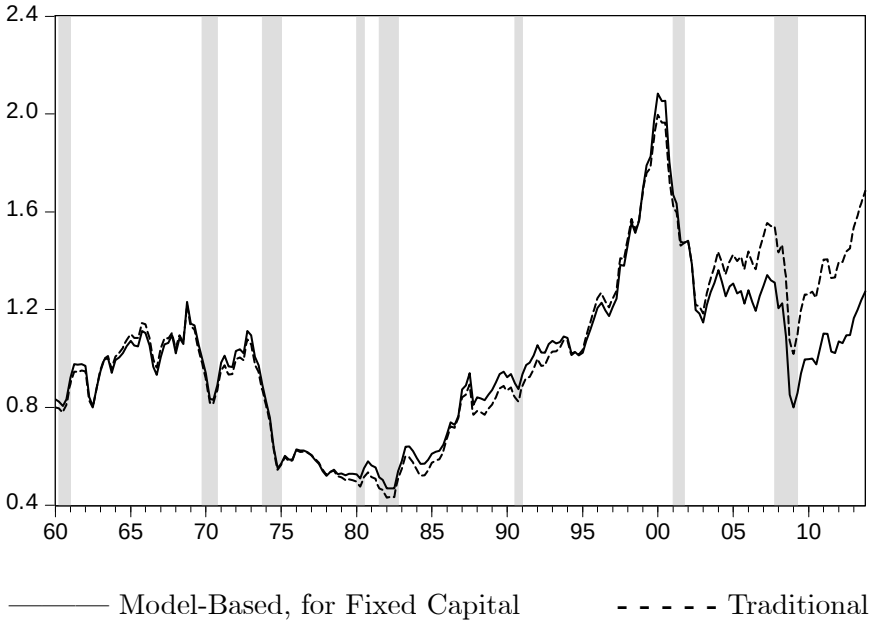


Table 1. Estimated Coefficients from the Investment Equations

	α^* from Kalman filter		Constant α^*	
	Estimate	z-statistic	Estimate	z-statistic
$c1$	1	n.a.	1	n.a.
$c2$	0.84	17	0.70	12
$c3$	0.28	21	0.28	n.a.
$c4$	0.20	16	0.18	15
$\sum_j \beta_{mj}$:				
Equipment	0.41		0.44	
Structures	0.87		0.73	
IPP	0.14		0.21	
$\beta_{inv,1}$	0.04	1	0.05	1
Average lag of β_{mj}				
Equipment	6		6	
Structures	10		10	
IPP	7		10	
Average lag of γ_{mk}				
Equipment	11		13	
Structures	14		14	
IPP	14		14	
α^*	Variable		0.27	

Notes: $c1$ is the coefficient of the actual tax treatment of revenue. $c2$ is the coefficient of the actual tax treatment of new capital. $c3$ is the coefficient of a dummy for 1981-1986 in the structures equation. $c4$ is the coefficient of the market value of capital. The β_{mj} are the coefficients of the change in output divided by cyclically adjusted output per worker ($Y/\bar{y}$). IPP is intellectual property products. $\beta_{inv,1}$ is the coefficient of the contemporaneous change in $Y/\bar{y}$ in the inventory equation. Average lags are in quarters. The γ_{mk} are the coefficients of the change in full employment ($\bar{N}$).

Table 2. Sensitivity Tests

	$\delta_I = 10$		$\widehat{rpy} = 4.5$		Standard error of $\alpha^* = 0.001$	
	Estimate	z-stat	Estimate	z-stat	Estimate	z-stat
$c1$	1	n.a.	1	n.a.	1	n.a.
$c2$	0.83	17	0.85	17	0.61	14
$c3$	0.29	20	0.30	21	0.20	18
$c4$	0.19	16	0.19	15	0.21	17
$\sum_j \beta_{mj}$:						
Equipment	0.41		0.40		0.38	
Structures	0.87		0.87		0.81	
IPP	0.14		0.13		0.11	
$\beta_{inv,1}$	0.04	1	0.05	1	0.04	1

Notes: δ_I is the service life of unmeasured intangible capital. $\widehat{rpy}$ is the sample average of the real cost of debt and equity less expected productivity growth.